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ON INTUITIONISTIC FUZZY CONTRA REGULAR α GENERALIZED CONTINUOUS MAPPINGS

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ABSTRACT
The purpose of this paper is to introduce the notion of intuitionistic fuzzy contra regular α generalized continuous mappings and study their behaviour and properties in intuitionistic fuzzy topological spaces. Additionally we obtain some interesting theorems.

I. INTRODUCTION
Zadeh [13] introduced the notion of fuzzy sets. After which there have been a number of generalizations on this fundamental concept. Chang [2] proposed fuzzy topology in 1967. The notion of intuitionistic fuzzy sets introduced by Atanassov [1] is one among them. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy topological space. In 2010, K. Sakthivel [11] introduced intuitionistic fuzzy alpha generalized continuous mappings and intuitionistic fuzzy regular α generalized continuous mappings was introduced by Nivetha M and Jayanthi D [10]. In this paper we introduce the notion of intuitionistic fuzzy contra regular α generalized continuous mappings and study their behaviour and properties in intuitionistic fuzzy topological spaces. Additionally we obtain some interesting theorems.

II. PRELIMINARIES

Definition 2.1:[1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ where the function $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2: [1] Let A and B be two IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in X\}$. Then

a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
c) $A^c = \{\langle x, , \nu_A(x), \mu_A(x)\rangle / x \in X\}$
d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$
e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X\}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$. The IFS $0\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1\sim = \{\langle x, 1, 0\rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFS in X satisfying the following axioms:

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a) $0\sim$, $1\sim \in \tau$
b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
c) $\cup G_i \in \tau$ for any family $\{ G_i / i \in J\} \subseteq \tau$

In this case the pair (X,τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in (X,τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [3] Let (X,τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$int(A) = \cup \{ G / G$ is an IFOS in X and $G \subseteq A\}$

$cl(A) = \cap \{K / K$ is an IFCS in X and $A \subseteq K\}$

Note that for any IFS A in (X,τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Corollary 2.5:[3] Let A, $A_i(i\in J)$ be intuitionistic fuzzy sets in X and B, $B_j(j\in K)$ be intuitionistic fuzzy sets in Y and $f : X\rightarrow Y$ be a function.
Then

a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$
b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$
c) $A \subseteq f^{-1}(f(A))$ [If f is injective , then $A=f^{-1}(f(A))$]
d) $f(f^{-1}(B)) \subseteq B$ [If f is surjective , then $B=f(f^{-1}(B))$]
e) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$
f) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$
g) $f^{-1}(0\sim) = 0\sim$
h) $f^{-1}(1\sim) = 1\sim$
i) $f^{-1}(B^c) = (f^{-1}(B))^c$

Definition 2.6:[5] An IFS A in an IFTS (X,τ) is said to be an

a) intuitionistic fuzzy semi closed set (IFSCS in short) if $int(cl(A)) \subseteq A$
b) intuitionistic fuzzy α closed set (IFαCS in short) if $cl(int(cl(A))) \subseteq A$
c) intuitionistic fuzzy pre closed set (IFPCS in short) if $cl(int(A)) \subseteq A$
d) intuitionistic fuzzy regular closed set (IFRCS in short) if $cl(int(A)) = A$

Definition 2.7:[8] An IFS A of an IFTS (X,τ) is called an intuitionistic fuzzy regular α generalized closed set (IFRαGCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X.

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Definition 2.8:[9] An IFS A of an IFTS (X,τ) is called an intuitionistic fuzzy regular α generalized open set (IFRαGOS in short) if αint(A) ⊇ U whenever A ⊇ U and U is an IFRCS in X.

The family of all IFRαGOSs of an IFTS (X,τ) is denoted by IFRαGO(X).

Definition 2.9:[12] Two IFSs A and B are said to be q-coincident ($A_q B$ in short) if and only if there exists an element x ∈ X such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.10:[4] An intuitionistic fuzzy point (IFP in short), written as $p_{(\alpha,\beta)}$, is defined to be an IFS of X given by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha,\beta) & \text{if } x = p, \\ (0,1) & \text{otherwise.} \end{cases}$$

An IFP $p_{(\alpha,\beta)}$ is said to belong to a set A if $\alpha \le \mu_A$ and $\beta \ge \nu_A$

Definition 2.11:[5] Let f be a mapping from an IFTS (X,τ) into an IFTS (Y,σ). Then f is said to be an intuitionistic fuzzy continuous (IF continuous in short) mapping if $f^{-1}(B)$ ∈ IFO(X) for every B ∈ σ.

Definition 2.12:[12] Let f be a mapping from an IFTS (X,τ) into an IFTS (Y,σ). Then f is said to be an intuitionistic fuzzy generalized continuous (IFG continuous in short) mapping if $f^{-1}(B)$ ∈ IFGCS(X) for every IFCS B in Y.

Definition 2.13:[9] If every IFRαGCS in (X,τ) is an IFαCS in (X,τ), then the space can be called as an intuitionistic fuzzy regular α $T_{1/2}$ ($IF_{r\alpha}T_{1/2}$ in short) space.

Definition 2.14:[9] An IFTS (X,τ) is said to be an intuitionistic fuzzy regular α $T^*_{1/2}$ ($IF_{r\alpha}T^*_{1/2}$ in short) space if every IFRαGCS is an IFCS in (X,τ).

Definition 2.15:[9] An IFTS (X,τ) is said to be an intuitionistic fuzzy regular α generalized $T_{1/2}$ ($IF_{r\alpha g}T_{1/2}$ in short) space if every IFRαGCS in X is an IFαGCS in X.

Definition 2.16:[7] Let f be a mapping from an IFTS (X,τ) into an IFTS (Y,σ). Then f is said to be an

(i) intuitionistic fuzzy contra continuous mapping (IFC continuous mapping in short) if $f^{-1}(B)$ ∈ IFO(X) for each IFCS B in Y
(ii) intuitionistic fuzzy contra α- continuous mapping (IFCα continuous mapping in short) if $f^{-1}(B)$ ∈ IFαO(X) for each IFCS B in Y
(iii) intuitionistic fuzzy contra pre continuous mapping (IFCP continuous mapping in short) if $f^{-1}(B)$ ∈ IFPO(X) for each IFCS B in Y

III. INTUITIONISTIC FUZZY CONTRA REGULAR α GENERALIZED CONTINUOUS MAPPINGS

In this section we introduce intuitionistic fuzzy contra regular α generalized continuous mapping and investigate some of its properties.

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Definition 3.1: A mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ is called an intuitionistic fuzzy contra regular α generalized continuous (IFCRαG continuous in short) mapping if $f^{-1}(A)$ is an IFRαGCS in (X,τ) for every IFOS A of (Y,σ).

Example 3.2: Let X = {a,b}, Y = {u,v} and $G_1 = \langle x, (0.4,0.4), (0.2,0.2)\rangle$ where μ_a=0.4, μ_b=0.4, ν_a=0.2, ν_b=0.2 and $G_2 = \langle x, (0.2,0.2), (0.7,0.7)\rangle$ where μ_a=0.2, μ_b=0.2, ν_a=0.7, ν_b=0.7 and $G_3 = \langle y, (0.2,0.1), (0.7,0.7)\rangle$ where μ_u=0.2, μ_v=0.1, ν_u=0.7, ν_v=0.7. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ by f(a) = u and f(b) = v. The IFS $G_3 = \langle y, (0.2,0.1), (0.7,0.7)\rangle$ is an IFOS in Y. Then $f^{-1}(G_3) = \langle x, (0.2,0.1), (0.7,0.7)\rangle$ where μ_a=0.2, μ_b=0.1, ν_a=0.7, ν_b=0.7 is an IFS in X. Then $f^{-1}(G_3) \subseteq G_1$ where G_1 is an IFROS in X. Now $\alpha cl(f^{-1}(G_3)) = G_1^c \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFRαGCS in X. Thus f is an IFCRαG continuous mapping.

Remark 3.3: Every IFC continuous mapping and IFαC continuous mapping are IFCRαG continuous mapping but the converses are not true in general. This can be seen from the following examples.

Example 3.4: Let X = {a,b}, Y = {u,v} and $G_1 = \langle x, (0.3,0.3), (0.2,0.2)\rangle$, $G_2 = \langle x, (0.2,0.2), (0.7,0.7)\rangle$ and $G_3 = \langle y, (0.1,0.1), (0.6,0.6)\rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ by f(a) = u and f(b) = v. The IFS $G_3 = \langle y, (0.1,0.1), (0.6,0.6)\rangle$ is an IFOS in Y. Then $f^{-1}(G_3) = \langle x, (0.1,0.1), (0.6,0.6)\rangle$ is an IFS in X. Then $f^{-1}(G_3) \subseteq G_1$ where G_1 is an IFROS in X. Now $\alpha cl(f^{-1}(G_3)) = G_1^c \subseteq G_1$. Therefore $f^{-1}(G_3)$ is an IFRαGCS in X but not an IFCS in X, since $cl(f^{-1}(G_3)) = G_1^c \neq f^{-1}(G_3)$. Therefore f is an IFCRαG continuous mapping but not an IFC continuous mapping.

Example 3.5: Let X = {a,b}, Y = {u,v} and $G_1 = \langle x, (0.5,0.4), (0.2,0.2)\rangle$, $G_2 = \langle x, (0.2,0.2), (0.7,0.7)\rangle$ and $G_3 = \langle y, (0.2,0.1), (0.6,0.6)\rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ by f(a) = u and f(b) = v. The IFS $G_3 = \langle y, (0.2,0.1), (0.6,0.6)\rangle$ is an IFOS in Y. Then $f^{-1}(G_3) = \langle x, (0.2,0.1), (0.6,0.6)\rangle$ is an IFS in X. Then $f^{-1}(G_3) \subseteq G_1$ where G_1 is an IFROS in X. Now $\alpha cl(f^{-1}(G_3)) = G_1^c \subseteq G_1$. Therefore $f^{-1}(G_3)$ is an IFRαGCS in X but not an IFαCS in X, since $cl(int(cl(f^{-1}(G_3)))) = G_1^c \not\subseteq f^{-1}(G_3)$. Therefore f is an IFCRαG continuous mapping but not an IFCα continuous mapping.

Remark 3.6: IFCP continuous mapping and IFCRαG continuous mapping are independent to each other.

Example 3.7: Let X = {a,b}, Y = {u,v} and $G_1 = \langle x, (0.4,0.3), (0.1,0.1)\rangle$, $G_2 = \langle x, (0.1,0.1), (0.7,0.7)\rangle$ and $G_3 = \langle y, (0.1,0.1), (0.6,0.7)\rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ by f(a) = u and f(b) = v. The IFS $G_3 = \langle y, (0.1,0.1), (0.6,0.7)\rangle$ is an IFOS in Y. Then $f^{-1}(G_3) = \langle x, (0.1,0.1), (0.6,0.7)\rangle$ is an IFS in X. Then $f^{-1}(G_3) \subseteq G_1$ where G_1 is an IFROS in X. Now $\alpha cl(f^{-1}(G_3)) = G_1^c \subseteq G_1$. Therefore $f^{-1}(G_3)$ is an IFRαGCS in X but not an IFPCS in X, since $cl(int(f^{-1}(G_3))) = G_1^c \not\subseteq f^{-1}(G_3)$. Therefore f is an IFCRαG continuous mapping but not an IFCP continuous mapping.

Example 3.8: Let X = {a,b}, Y = {u,v} and $G_1 = \langle x, (0.7,0.6), (0.2,0.2)\rangle$, $G_2 = \langle x, (0.1,0.2), (0.7,0.6)\rangle$ and $G_3 = \langle y, (0.1,0.2), (0.8,0.8)\rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ by f(a) = u and f(b) = v. The IFS $G_3 = \langle y, (0.1,0.2), (0.8,0.8)\rangle$ is an IFOS in Y. Then $f^{-1}(G_3) = \langle x, (0.1,0.2), (0.8,0.8)\rangle$ is an IFS in X. Now $cl(int(f^{-1}(G_3))) = 0\sim \subseteq f^{-1}(G_3)$. Therefore $f^{-1}(G_3)$ is an IFPCS in X but not an IFRαGCS in X, since $\alpha cl(f^{-1}(G_3)) = G_1^c \not\subseteq G_2$ but $f^{-1}(G_3) \subseteq G_2$ where G_2 is an IFROS in X. Therefore f is an IFCP continuous mapping but not an IFCRαG continuous mapping.

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Remark 3.9: IFCS continuous mapping and IFCRαG continuous mapping are independent to each other.

Example 3.10: Let X = {a,b}, Y = {u,v} and $G_1 = \langle x, (0.5,0.5), (0.3,0.2)\rangle$,$G_2 = \langle x, (0.1,0.2), (0.9,0.8)\rangle$ and $G_3 = \langle y, (0.3,0.1), (0.6,0.6)\rangle$. Then τ = {0~, G_1, G_2, 1~} and σ = {0~, G_3, 1~} are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y,σ) by f(a) = u and f(b) = v. The IFS $G_3 = \langle y, (0.3,0.1), (0.6,0.6)\rangle$ is an IFOS in Y. Then $f^{-1}(G_3) = \langle x, (0.3,0.1), (0.6,0.6)\rangle$ is an IFS in X. Then $f^{-1}(G_3) \subseteq G_1$ where G_1 is an IFROS in X. Now $\alpha cl(f^{-1}(G_3)) = G_1^c \subseteq G_1$. Therefore $f^{-1}(G_3)$ is an IFRαGCS in X but not an IFSCS in X, since $int(cl(f^{-1}(G_3))) = G_2 \nsubseteq f^{-1}(G_3)$. Therefore f is an IFCRαG continuous mapping but not an IFCS continuous mapping.

Example 3.11: Let X = {a,b}, Y = {u,v} and $G_1 = \langle x, (0.5,0.2), (0.5,0.7)\rangle$,$G_2 = \langle x, (0.1,0.1), (0.7,0.7)\rangle$ and $G_3 = \langle y, (0.5,0.2), (0.5,0.7)\rangle$. Then τ = {0~, G_1, G_2, 1~} and σ = {0~, G_3, 1~} are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y,σ) by f(a) = u and f(b) = v. The IFS $G_3 = \langle y, (0.5,0.2), (0.5,0.7)\rangle$ is an IFOS in Y. Then $f^{-1}(G_3) = \langle x, (0.5,0.2), (0.5,0.7)\rangle$ is an IFS in X. Now $int(cl(f^{-1}(G_3))) = f^{-1}(G_3)$ is an IFSCS in X but not an IFRαGCS in X, since $\alpha cl(f^{-1}(G_3)) = G_1^c \nsubseteq G_1$ but $f^{-1}(G_3) \subseteq G_1$ where G_1 is an IFROS in X. Therefore f is an IFCS continuous mapping but not an IFCRαG continuous mapping.

Theorem 3.12: A mapping f : X → Y is an IFCRαG continuous mapping if and only if the inverse image of each IFCS in Y is an IFRαGOS in X.

Proof: Necessity: Let A be an IFCS in Y. This implies A^c is an IFOS in Y. Since f is an IFCRαG continuous mapping, $f^{-1}(A^c)$ is an IFRαGCS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFRαGOS in X.

Sufficiency: Let A be an IFOS in Y. This implies A^c is an IFCS in Y. By hypothesis, $f^{-1}(A^c)$ is an IFRαGOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, where $(f^{-1}(A))^c$ is an IFRαGOS in X, $f^{-1}(A)$ is an IFRαGCS in X. Hence f is an IFCRαG continuous mapping.

Theorem 3.13: Let f : (X,τ) → (Y,σ) be a mapping and let $f^{-1}(A)$ be an IFROS in X for every IFCS A in Y. Then f is an IFCRαG continuous mapping.

Proof: Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFROS in X, by hypothesis. Since every IFROS is an IFRαGOS [9], $f^{-1}(A)$ is an IFRαGOS in X. Hence f is an IFCRαG continuous mapping, by Theorem 3.12.

Theorem 3.14: Let f : (X,τ) → (Y,σ) be an IFCRαG continuous mapping, then

(i) f is an IFC continuous mapping if X is an $IF_{r\alpha}T^*_{1/2}$ space
(ii) f is an IFCα continuous mapping if X is an $IF_{r\alpha}T_{1/2}$ space
(iii) f is an IFCαG continuous mapping if X is an $IF_{r\alpha g}T_{1/2}$ space

Proof: (i) Let A be an IFOS in Y. Then $f^{-1}(A)$ is an IFRαGCS in X, by hypothesis. Since X is an $IF_{r\alpha}T^*_{1/2}$ space, $f^{-1}(A)$ is an IFCS in X. Hence f is an IFC continuous mapping.

(ii) Let A be an IFOS in Y. Then $f^{-1}(A)$ is an IFRαGCS in X, by hypothesis. Since X is an $IF_{r\alpha}T_{1/2}$ space, $f^{-1}(A)$ is an IFαCS in X. Hence f is an IFCα continuous mapping.

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(iii) Let A be an IFOS in Y. Then $f^{-1}(A)$ is an IFRαGCS in X, by hypothesis. Since X is an $IF_{r\alpha g}T_{1/2}$ space, $f^{-1}(A)$ is an IFαGCS in X. Hence f is an IFCαG continuous mapping.

Theorem 3.15: Let $f : (X,\tau) \to (Y,\sigma)$ be an IFCRαG continuous mapping and $g : (Y,\sigma) \to (Z,\gamma)$ be an IF continuous mapping, then $g \circ f : (X,\tau) \to (Z,\gamma)$ is an IFCRαG continuous mapping.

Proof: Let A be an IFOS in Z. Then $g^{-1}(A)$ is an IFOS in Y, by hypothesis. Since f is an IFCRαG continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFRαGCS in X. Hence $g \circ f$ is an IFCRαG continuous mapping.

Theorem 3.16: Let $f : (X,\tau) \to (Y,\sigma)$ be an IFCRαG continuous mapping and $g : (Y,\sigma) \to (Z,\gamma)$ be an IFG continuous mapping and Y is an $IFT_{1/2}$ space, then $g \circ f : (X,\tau) \to (Z,\gamma)$ is an IFCRαG continuous mapping.

Proof: Let A be an IFOS in Z. Then $g^{-1}(A)$ is an IFGOS in Y, by hypothesis. Since Y is an $IFT_{1/2}$ space, $g^{-1}(A)$ is an IFOS in Y. Therefore $f^{-1}(g^{-1}(A))$ is an IFRαGCS in X, by hypothesis. Hence $g \circ f$ is an IFCRαG continuous mapping.

Theorem 3.17: Let $f : X \to Y$ be a mapping. Suppose that one of the following properties hold:

(i) $f(\alpha cl(A)) \subseteq int(f(A))$ for each IFS A in X
(ii) $\alpha cl(f^{-1}(B)) \subseteq f^{-1}(int(B))$ for each IFS B in Y
(iii) $f^{-1}(cl(B)) \subseteq \alpha int(f^{-1}(B))$ for each IFS B in Y

Then f is an IFCRαG continuous mapping.

Proof: (i) ⇒ (ii) Let B be an IFS in Y. Then $f^{-1}(B)$ is an IFS in X. By hypothesis, $f(\alpha cl(f^{-1}(B))) \subseteq int(f(f^{-1}(B))) \subseteq int(B)$. Now $\alpha cl(f^{-1}(B)) \subseteq f^{-1}(f(\alpha cl(f^{-1}(B)))) \subseteq f^{-1}(int(B))$

(ii) ⇒ (iii) is obvious by taking complement in (ii).

Suppose (iii) holds: Let A be an IFCS in Y. Then cl(A) = A and $f^{-1}(A)$ is an IFS in X. Now $f^{-1}(A) = f^{-1}(cl(A)) \subseteq \alpha int(f^{-1}(A)) \subseteq f^{-1}(A)$, by hypothesis. This implies $f^{-1}(A)$ is an IFαOS in X and hence an IFRαGOS[9] in X. Thus f is an IFCRαG continuous mapping.

Theorem 3.18: Let $f : X \to Y$ be a bijective mapping. Then f is an IFCRαG continuous mapping if $cl(f(A)) \subseteq f(\alpha int(A))$ for every IFS A in X.

Proof: Let A be an IFCS in Y. Then cl(A) = A and $f^{-1}(A)$ is an IFS in X. By hypothesis $cl(f(f^{-1}(A))) \subseteq f(\alpha int(f^{-1}(A)))$. Since f is an onto, $f(f^{-1}(A)) = A$. Therefore $A = cl(A) = cl(f(f^{-1}(A))) \subseteq f(\alpha int(f^{-1}(A)))$. Now $f^{-1}(A) \subseteq f^{-1}(f(\alpha int(f^{-1}(A)))) = \alpha int(f^{-1}(A)) \subseteq f^{-1}(A)$. Hence $f^{-1}(A)$ is an IFαOS in X and hence an IFRαGOS[9] in X. Thus f is an IFCRαG continuous mapping.

Theorem 3.19: If $f : X \to Y$ is an IFCRαG continuous mapping, where X is an $IF_{r\alpha}T_{1/2}$ space, then the following conditions hold:

(i) $\alpha cl(f^{-1}(B)) \subseteq f^{-1}(int(\alpha cl(B)))$ for every IFOS B in Y
(ii) $f^{-1}(cl(\alpha int(B))) \subseteq \alpha int(f^{-1}(B))$ for every IFCS B in Y

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Proof: (i) Let B be an IFOS in Y. By hypothesis $f^{-1}(B)$ is an IFRαGCS in X. Since X is an $IF_{r\alpha}T_{1/2}$ space, $f^{-1}(B)$ is an IFαCS in X. This implies $\alpha cl(f^{-1}(B)) = f^{-1}(B) = f^{-1}(int(B) \subseteq f^{-1}(int(\alpha cl(B)))$.

(ii) can be proved easily by taking the complement of (i).

Theorem 3.20: If $f : X \rightarrow Y$ is a mapping where X is an $IF_{r\alpha}T_{1/2}$ space, then the following are equivalent:

(i) f is an IFCRαG continuous mapping

(ii) for each IFP $p_{(\alpha,\beta)} \in X$ and for each IFCS B containing $f(p_{(\alpha,\beta)})$, there exists an IFαOS $A \subseteq X$ and $p_{(\alpha,\beta)} \in A$ such that $A \subseteq f^{-1}(B)$

(iii) for each IFP $p_{(\alpha,\beta)} \in X$ and for each IFCS B containing $f(p_{(\alpha,\beta)})$, there exists an IFαOS $A \subseteq X$ and $p_{(\alpha,\beta)} \in A$ such that $f(A) \subseteq B$

Proof: (i) ⇒ (ii) Let B be an IFCS in Y. Let $p_{(\alpha,\beta)}$ be an IFP in X such that $f(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(f(p_{(\alpha,\beta)})) \in f^{-1}(B)$. By hypothesis $f^{-1}(B)$ is an IFRαGOS in X. Since X is an $IF_{r\alpha}T_{1/2}$ space, $f^{-1}(B)$ is an IFαOS in X. Now let $A = \alpha int(f^{-1}(B)) \subseteq f^{-1}(B)$. Therefore $A \subseteq f^{-1}(B)$.

(ii) ⇒ (iii) Let B be IFCS in Y. Let $p_{(\alpha,\beta)}$ be an IFP in X such that $f(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(f(p_{(\alpha,\beta)})) \in f^{-1}(B)$. By hypothesis $f^{-1}(B)$ is an IFαOS in X and $A \subseteq f^{-1}(B)$. This implies $f(A) \subseteq f(f^{-1}(B)) \subseteq B$.

(iii) ⇒ (i) Let B be any IFCS in Y and let $p_{(\alpha,\beta)} \in X$. Let $f(p_{(\alpha,\beta)}) \in B$. By hypothesis there exists an IFαOS A in X such that $p_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$. This implies $p_{(\alpha,\beta)} \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$. That is $p_{(\alpha,\beta)} \in f^{-1}(B)$. Since A is an IFαOS, $A = \alpha int(A) \subseteq \alpha int(f^{-1}(B))$. Therefore $p_{(\alpha,\beta)} \in \alpha int(f^{-1}(B))$. But $f^{-1}(B) = \cup_{p(\alpha,\beta) \in f^{-1}(B)} p_{(\alpha,\beta)} \subseteq \alpha int(f^{-1}(B)) \subseteq f^{-1}(B)$. Hence $f^{-1}(B)$ is an IFαOS in X and hence $f^{-1}(B)$ is an IFRαGOS[9] in X. Thus f is an IFCRαG continuous mapping.

Theorem 3.21: For a mapping $f : X \rightarrow Y$, the following are equivalent, where X is an $IF_{r\alpha}T_{1/2}$ space:

(i) f is an IFCRαG continuous mapping

(ii) for every IFCS A in Y, $f^{-1}(A)$ is an IFRαGOS in X

(iii) for every IFOS B in Y, $f^{-1}(B)$ is an IFRαGCS in X

(iv) for any IFCS A in Y and for any IFP $p_{(\alpha,\beta)} \in X$, if $f(p_{(\alpha,\beta)})\ _q\ A$, then $p_{(\alpha,\beta)}\ _q\ \alpha int(f^{-1}(A))$

(v) for any IFCS A in Y and for any IFP $p_{(\alpha,\beta)} \in X$, if $f(p_{(\alpha,\beta)})\ _q\ A$, then there exists an IFRαGOS B such that $p_{(\alpha,\beta)}\ _q\ B$ and $f(B) \subseteq A$

Proof: (i) ⇔ (ii) and (ii) ⇔ (iii) are obvious.

(ii) ⇒ (iv) Let $A \subseteq Y$ be an IFCS and let $p_{(\alpha,\beta)} \in X$. Let $f(p_{(\alpha,\beta)})\ _q\ A$. Then $p_{(\alpha,\beta)}\ _q\ f^{-1}(A)$. By hypothesis, $f^{-1}(A)$ is an IFRαGOS in X. Since X is an $IF_{r\alpha}T_{1/2}$ space, $f^{-1}(A)$ is an IFαOS in X. This implies $\alpha int(f^{-1}(A)) = f^{-1}(A)$. Hence $p_{(\alpha,\beta)}\ _q\ \alpha int(f^{-1}(A))$.

(iv) ⇒ (ii) Let $A \subseteq Y$ be an IFCS and let $p_{(\alpha,\beta)} \in X$. Let $f(p_{(\alpha,\beta)})\ _q\ A$. Then $p_{(\alpha,\beta)}\ _q\ f^{-1}(A)$. By hypothesis, $p_{(\alpha,\beta)}\ _q\ \alpha int(f^{-1}(A))$. That is $f^{-1}(A) \subseteq \alpha int(f^{-1}(A))$. But $\alpha int(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore $f^{-1}(A) = \alpha int(f^{-1}(A))$. Thus $f^{-1}(A)$ is an IFαOS in X and hence $f^{-1}(A)$ is an IFRαGOS[9] in X.

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(iv) ⇒ (v) Let $A \subseteq Y$ be an IFCS and let $p_{(\alpha,\beta)} \in X$. Let $f(p_{(\alpha,\beta)})\ _q\ A$. Then $p_{(\alpha,\beta)\ q}\ f^{-1}(A)$. By hypothesis, $p_{(\alpha,\beta)\ q}\ \alpha int(f^{-1}(A))$. Thus $f^{-1}(A)$ is an IFαOS in X and hence $f^{-1}(A)$ is an IFRαGOS[9] in X. Let $f^{-1}(A) = B$. Therefore $p_{(\alpha,\beta)\ q}\ B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

(v) ⇒ (iv) Let $A \subseteq Y$ be an IFCS and let $p_{(\alpha,\beta)} \in X$. Let $f(p_{(\alpha,\beta)})\ _q\ A$. Then $p_{(\alpha,\beta)\ q}\ f^{-1}(A)$. By hypothesis, there exists an IFRαGOS B in X such that $p_{(\alpha,\beta)\ q}\ B$ and $f(B) \subseteq A$. Let $B = f^{-1}(A)$. Since X is an $IF_{r\alpha}T_{1/2}$ space, $f^{-1}(A)$ is an IFαOS in X. Therefore $p_{(\alpha,\beta)\ q}\ \alpha int(f^{-1}(A))$.

Theorem 3.22: A mapping $f : X \to Y$ is an IFCRαG continuous mapping if $f^{-1}(\alpha cl(B)) \subseteq int(f^{-1}(B))$ for every IFS B in Y.

Proof: Let $B \subseteq Y$ be an IFCS. Then $cl(B) = B$. Since every IFCS is an IFαCS, $\alpha cl(B) = B$. Now by hypothesis, $f^{-1}(B) = f^{-1}(\alpha cl(B)) \subseteq int(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B) = int(f^{-1}(B))$. Therefore $f^{-1}(B)$ is an IFOS in X. Hence f is an IFC continuous mapping. Then by Remark 3.3, f is an IFCRαG continuous mapping.

Theorem 3.23: A mapping $f : X \to Y$ is an IFCRαG continuous mapping, where X is an $IF_{r\alpha}T_{1/2}$ space if and only if $f^{-1}(\alpha cl(B)) \subseteq \alpha int(f^{-1}(cl(B)))$ for every IFS B in Y.

Proof: Necessity: Let $B \subseteq Y$ be an IFS. Then cl(B) is an IFCS in Y. By hypothesis $f^{-1}(cl(B))$ is an IFRαGOS in X. Since X is an $IF_{r\alpha}T_{1/2}$ space, $f^{-1}(cl(B))$ is an IFαOS in X. This implies $f^{-1}(cl(B)) = \alpha int(f^{-1}(cl(B)))$. Therefore $f^{-1}(\alpha cl(B)) \subseteq f^{-1}(cl(B)) = \alpha int(f^{-1}(cl(B)))$.

Sufficiency: Let $B \subseteq Y$ be an IFS. Then cl(B) is an IFCS in Y. By hypothesis, $f^{-1}(\alpha cl(B)) \subseteq \alpha int(f^{-1}(cl(B))) = \alpha int(f^{-1}(B))$. But $\alpha cl(B) = B$. Therefore $f^{-1}(B) = f^{-1}(\alpha cl(B)) \subseteq \alpha int(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFαOS in X and hence $f^{-1}(B)$ is an IFRαGOS[9] in X. Hence f is an IFCRαG continuous mapping.

Theorem 3.24: An IF continuous mapping $f : X \to Y$ is an IFCRαG continuous mapping, if IFRαGO(X) = IFRαGC(X).

Proof: Let $A \subseteq Y$ be an IFOS. By hypothesis, $f^{-1}(A)$ is an IFOS in X and hence $f^{-1}(A)$ is an IFRαGOS[9] in X. Thus $f^{-1}(A)$ is an IFRαGCS in X, as IFRαGO(X) = IFRαGC(X). Therefore f is an IFCRαG continuous mapping.

IV. CONCLUSION

Thus we have analyzed relationship between intuitionistic fuzzy regular α generalized contra continuous mapping and the already existing intuitionistic fuzzy continuous mappings and obtain many interesting theorem concern with the intuitionistic fuzzy regular α generalized contra continuous mapping

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