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### AN OVERVIEW OF FUZZY MATHEMATICS AND FUZZY LOGIC

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#### ABSTRACT

The term Fuzzy mathematics is related to research and treatment of phenomenon of ambiguity. Fuzziness can be found in almost all areas of daily life, for example in statements, engineering, controller designing for a plant or a process, manufacturing, optimization and economy etc. It is however, particularly faced in all the areas in which human evaluation, understanding, judgement and decisions are required. Fuzzy concepts are some form of generalizations of mathematical concepts. Probability and possibility are the two complementary theories. The fuzzy principle states that everything is a matter of degree of membership. In terms of mathematics, fuzzy sets are sets whose elements have degrees of membership. Many real-world problems cannot be described and handled by the use of classical set theory and it includes all those involving elements with only partial membership of a set. Fuzzy set theory, on the other side, accepts partial memberships, and, therefore, in some sense it generalizes the classical set theory to some extent. The fuzzy set theory is a very natural extension of the classical set theory, and also is a rigorous mathematical notion.

#### INTRODUCTION

The classical set theory is built on the fundamental concept of “set” of which an individual is either a member or not a member. A sharp, crisp, and unambiguous distinction exists between a member and a non member for any well-defined “set”. There is a very clear, precise or sharp boundary to indicate whether an element belongs to the set? When one asks the question “Is this element a member of that set?” The answer is either “yes” or “no.” This is true for both the deterministic and the stochastic cases. In probability and statistics, one may ask a question like “What is the probability of this element being a member of that set?” In this case, although an answer could be like “The probability for this entity to be a member of that set is 90%,” the final outcome (i.e., conclusion) is still either “it is” or “it is not” a member of the set. The chance for one to make a correct prediction as “it is a member of the set” is 90%, which does not mean that it has 90% membership in the set and in the meantime it possesses 10% non-membership. In the classical set theory, it is not allowed that an element is in a set and not in the set at the same time. Thus, many real-world application problems cannot be described and handled by the classical set theory, including all those involving elements with only partial membership of a set. On the contrary, fuzzy set theory accepts partial memberships, and, therefore, in a sense generalizes the classical set theory to some extent. The fuzzy set theory is a very natural extension of the classical set theory, and is also a rigorous mathematical notion.

#### FUZZY MATHEMATICS AND FUZZY LOGIC

Fuzzy mathematics forms a branch of mathematics which is related with fuzzy set theory and fuzzy logic. Fuzzy sets, as an extension of the classical notion of set, were introduced by Lotfi Asker Zadeh and Dieter Klaua in 1965. At the same time, Sali defined L-relations which are more general structures. In terms of mathematics, the elements of fuzzy sets have degrees of membership. In the case of classical set theory, the membership of elements in a set is assessed by Yes or No according to a possible bivalent condition — an element either belongs to a set or it does not belong to a set. Fuzzy set theory, on the other hand, allows a gradual assessment of the membership of elements in a set; this is described with the help of a membership function valued in the real interval  $[0, 1]$ . Fuzzy sets generalize classical sets, since the indicator functions of classical sets are special cases of the membership functions of fuzzy sets, if the latter only take values 0 or 1. In fuzzy set theory, classical bivalent sets are usually called crisp sets. The fuzzy set theory can be used in a wide range of areas in which the available information is incomplete or imprecise: such as bioinformatics, engineering, controller designing for any process control etc. Fuzzy relations, which are used now in different areas, such as linguistics, decision-making, and clustering are special cases of L-relations where L is the unit interval  $[0, 1]$ . A fuzzy subset A of a set X is a function  $A : X \rightarrow L$ , where L is the interval  $[0, 1]$ . This function is also called a membership function. A membership function is a generalization of a characteristic function or an indicator function of a subset defined for  $L = \{0, 1\}$ . More generally, one can use a complete lattice L in a definition of a fuzzy subset A. There are three stages of the evolution of the fuzzification of

mathematical concepts and these are: during the sixties and seventies, straightforward fuzzification, during the eighties, the explosion of the possible choices in the generalization process and in the nineties, the standardization, axiomatization and L-fuzzification. Usually, a fuzzification of mathematical concepts is based on a generalization of these concepts from characteristic functions to membership functions. Let  $A$  and  $B$  be two fuzzy subsets of  $X$ . Intersection  $A \cap B$  and union  $A \cup B$  are defined as follows:  $(A \cap B)(x) = \min(A(x), B(x))$ ,  $(A \cup B)(x) = \max(A(x), B(x))$  for all  $x \in X$ . Instead of min and max one can use t-norm and t-conorm, respectively, for example,  $\min(a, b)$  can be replaced by multiplication  $ab$ . A straightforward fuzzification is usually based on min and max operations because in this case more properties of traditional mathematics can be extended to the fuzzy case. A very important generalization principle used in fuzzification of algebraic operations is a closure property. Let  $*$  be a binary operation on  $X$ . The closure property for a fuzzy subset  $A$  of  $X$  is that for all  $x, y \in X$ ,  $A(x * y) \geq \min(A(x), A(y))$ . Let  $(G, *)$  be a group and  $A$  a fuzzy subset of  $G$ . Then  $A$  is a fuzzy subgroup of  $G$  if for all  $x, y$  in  $G$ ,  $A(x * y^{-1}) \geq \min(A(x), A(y^{-1}))$ . A similar generalization principle is used, for example, for fuzzification of the transitivity property. Let  $R$  be a fuzzy relation in  $X$ , i.e.  $R$  is a fuzzy subset of  $X \times X$ . Then  $R$  is transitive if for all  $x, y, z$  in  $X$ ,  $R(x, z) \geq \min(R(x, y), R(y, z))$ .

## CONCLUSION

Fuzzy logic is different from probability theory. Probability measures the likelihood of an event before the actual occurrence and the actual outcome is known to us, whereas the fuzzy logic measures the degree to which an outcome belongs to an event. Fuzzy logic and probability theory can be complementary and can be used to solve a real world problem. The behaviour of a fuzzy system is deterministic. There is nothing fuzzy, ambiguous, or mysterious about a fuzzy system, despite its name. Fuzzy logic differs from multivalued logic by introducing the concepts of linguistic variables and hedges that are important for fuzzy logic's agenda to capture human linguistic reasoning.

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