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NEW CONTINUOUS WAVELET FAMILY

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ABSTRACT

This paper proposes a new continuous wavelet family. Here we will take one function and those successive derivatives will become continuous by fundamental theorem of calculus .We will show that these functions become a new continuous wavelet family by satisfying the conditions of the wavelets without FIR filters and without scaling function like Mexican and Morlet. Wavelet analysis has attracted attention for its ability to analyze rapidly changing transient signals. Any application using the Fourier transform can be formulated using wavelets to provide more accurately localized temporial and frequency information.

Keywords: Admissibility condition, Continuous wavelets, Mother wavelet, Mexican and Morlet wavelets, Null moments..

INTRODUCTION

Wavelets have created much excitement in the mathematics community (perhaps more so than in engineering) because the mathematical development has followed a very interesting path. The recent developments can be viewed as resolving some of the difficulties inherent in Fourier analysis [1]. For example, a typical question is how to relate the Fourier coefficients to the global or local behavior of a function. The development of wavelet analysis can be considered an outgrowth of the Littlewood-Paley theory (first published in 1931), which sought a new approach to answer some of these difficulties. Again it is unifying framework made possible by recent results in Wavelet theory related to problems of Harmonic analysis (also to similar problems in Operator theory called the Calderon- Zygmud theory) that has generated much of the excitement. In signal processing Continuous Wavelet Transform (CWT) is very efficient in determining the damping ratio of oscillating signals (e.g. identification of damping in dynamical systems). CWT is also very resistant to the noise in the signal. Here we introduce new continuous wavelet family which also gives better results like Morlet and Mexican wavelets.

THE CONTINUOUS WAVELET TRANSFORMATION

$$CWT_X^{\psi}(\tau,S) = \frac{1}{\sqrt{s}}\int X(t)\psi\left(\frac{t-\tau}{s}\right)dt$$

As seen in the above equation, the transformed signal is a function of two variables [2], τ and s, the translation and scale parameters, respectively. $\psi(t)$ is the transforming function, and it is called the Mother wavelet. If the signal has a spectral component that corresponds to the value of s, the product of the wavelet with the signal at the location where this spectral components exists gives a relatively large value.

CONTINUOUS WAVELETS

Continuous wavelets are functions used by the Continuous wavelet transform [3]. These functions are defined as analytical expressions, as functions either of time or of frequency. Most of the continuous wavelets are used for both wavelet decomposition and composition transforms. The continuous wavelet transform (CWT) is used to analyze how the frequency content of a signal changes over time. For two signals, wavelet coherence reveals common time-varying patterns. For images, continuous wavelet analysis shows how the frequency content of an image varies across the image and helps to reveal patterns in a noisy image.

MEXICAN WAVELET

In mathematics and numerical analysis, the Mexican wavelet [4], i.e.

$$\psi(t) = (1-x^2)e^{-x^2/2}$$

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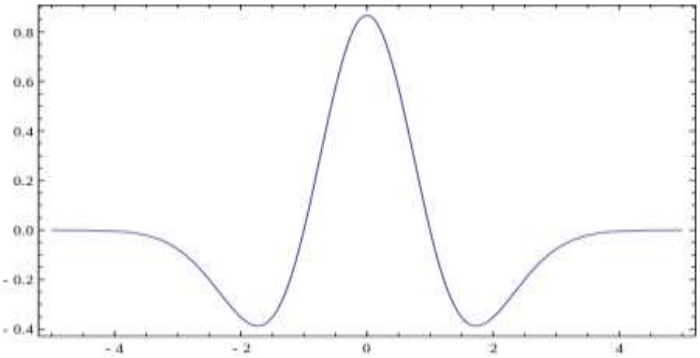
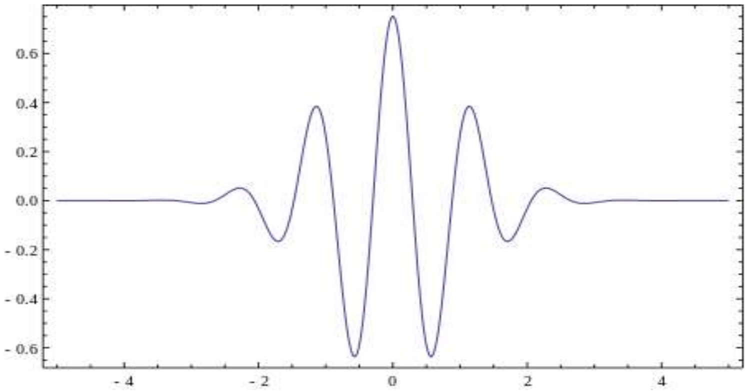


is the negative normalized second derivative of a Gaussian function,

MORLET WAVELET

The Morlet wavelet [5] (or Gabor wavelet) is a wavelet composed of cos(5x) multiplied by a Gaussian window (envelope). This wavelet is closely related to human perception, both hearing and vision.

$$\psi(t) = cos(5x)e^{-x^2}$$



MOTHER WAVELET

For practical applications and for efficiency reasons, one prefers continuously differentiable functions with compact support as Mother (prototype) wavelet (functions). However, to satisfy analytical requirements (in the continuous WT) and in general for theoretical reasons, one chooses the wavelet functions from the space L^2. This is the space of absolutely and square integrable.

THE MOTHER WAVELET MUST SATISFY FOLLOWING CONDITIONS:

1) Finite energy property

$$C_K = \int_{-\infty}^{\infty} |\psi(t)|^2 \, dt < \infty$$

2) Zero mean property

$\int_{-\infty}^{\infty} \psi(t)dt = 0$ and

3) The admissibility condition for the Mother wavelet

For the continuous wavelet transform to be invertible [6], the Mother wavelet $\psi(t)$ must satisfy the admissibility condition [6].

$$C_\psi = \int_0^{\infty} \frac{|\Psi(\omega)|^2}{|\omega|} d\omega < \infty$$

Where $\Psi(\omega)$ is the Fourier transformation of the $\psi(t)$.

The Fourier transformation is calculated by the formula below

$$\Psi(\omega) = (\frac{1}{2\Pi}) \int_{-\infty}^{\infty} \psi(t)e^{-j\omega t} \, dt.$$

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Then the admissibility condition implies that $\Psi(0) = 0$, that is, $\int \psi(t)dt = 0$. If $\Psi(0) \neq 0$,then from continuity, there is a small interval I containing 0, and $\epsilon > 0$. Such that $|\Psi(\omega)| > \epsilon$ for all ω in I. But it would then follow that

$$\int_0^\infty \frac{|\Psi(\omega)|^2}{|\omega|} d\omega \geq \int \frac{|\Psi(\omega)|^2}{|\omega|} d\omega \geq \int (\epsilon^2 /|\omega|) d\omega = \infty \text{ in I.}$$

The admissibility condition therefore implies that the Mother wavelet has no discontinuity component, i.e $\Psi(0) = 0$. One computes directly that

$$\Psi(0) = (\frac{1}{2\Pi}) \int_{-\infty}^{\infty} \psi(t)\, dt$$

And also the admissibility condition implies that the integral of the Mother wavelet is zero. For this to occur, the Mother wavelet must contain oscillations: it must have sufficient negative area to cancel out the positive area. Of course, this is what it means to have no discontinuity component.

THE NEW CONTINUOUS WAVELET FAMILY

Here we took function as f(t) = 1 / $(4+t^2)$, whose successive derivatives are represented by $\psi_1, \psi_2, \psi_3, \psi_4$, ψ_5 and ψ_6 are satisfying all the conditions of wavelets including admissibility conditions [9]. So these are becoming new continuous wavelet family. This family contains 6 members, and those functions are clearly mentioned below

1. $\psi_1 = -(2t)/(t^2 + 4)^2$
2. $\psi_2 = (8t^2)/(t^2 + 4)^3 - 2/(t^2 + 4)^2$
3. $\psi_3 = (24t)/(t^2 + 4)^3 - (48t^3)/(t^2 + 4)^4$
4. $\psi_4 = 24/(t^2 + 4)^3 - (288t^2)/(t^2 + 4)^4 + (384t^4)/(t^2+ 4)^5$
5. $\psi_5 = (3840t^3)/(t^2 + 4)^5 - (720t)/(t^2 + 4)^4 - (3840t^5)/(t^2 + 4)^6$
6. $\psi_6 = (17280t^2)/(t^2 + 4)^5 - 720/(t^2 + 4)^4 - (57600t^4)/(t^2 + 4)^6 + (46080t^6)/(t^2 + 4)^7$

These Continuous wavelet family here onwards named as Annavaram wavelet family. These wavelets also have null moments, the condition for null moments is

a) $\int_{-\infty}^{\infty} t^n \psi(t) dt = 0$ where n = 1, 2, 3…..

The first wavelet has zero null moments, second derivative has one null moment, third wavelet has two null moments, fourth wavelet has three null moments, fifth wavelet has four null moments and sixth wavelet has five null moments. We can proceed for further derivatives, which will also form wavelets, but for better applications we stopped at sixth derivative.

TABLE FOR C_K AND C_Ψ VALUES OF WAVELETS

S.NO	Wavelet	C_k	C_ψ	No. of null moments
1	$\psi_1(t)$	0.0245	-0.1542	0
2	$\psi_2(t)$	0.0184	0.0578	1
3	$\psi_3(t)$	0.0345	-0.0723	2
4	$\psi_4(t)$	0.1208	0.1898	3
5	$\psi_5(t)$	0.6795	-0.8539	4
6	$\psi_6(t)$	5.6059	5.8705	5

The plots of the above wavelets are [9]

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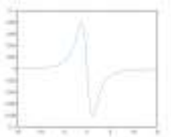


Fig (1)

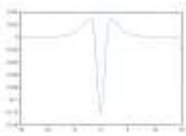


Fig (2)

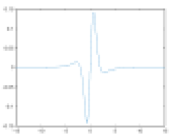
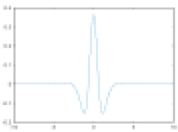


Fig (4)


Fig(3)

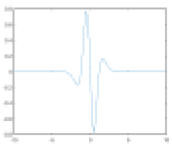


Fig (5)

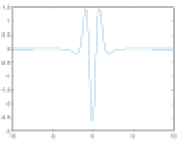


Fig (6)

CONCLUSION

The considered function f(t) whose derivatives are mentioned above will satisfy all the conditions of wavelets and constant values are also mentioned in the above table. This is the new continuous wavelet family of six members, whatever applications are there for other continuous wavelets, this family can also be used for those applications. Future scope of this paper is, there are many applications of continuous wavelet transform, just we have to apply this wavelet family in transformation using Matlab software to see the better results.

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