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A COMPARISON BETWEEN WEIBULL AND POWER LAW MODEL IN ESTIMATING CONDITIONAL INTENSITY FUNCTION FOR RAINFALL DATA

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ABSTRACT
In this paper, we study the conditional intensity function (hazard function) of Weibull and Power law model for rainfall data. These models are fitted to the daily rainfall data from 2012 to 2016 obtained from Ngurah Rai station, located in Denpasar Indonesia. The maximum likelihood estimation is used to estimate the parameters of the model. The result shows that the determination of the value of the conditional intensity on a daily rainfall data indicate that the use of Weibull and Power Law models for $\alpha < 1$ provide the same general characteristics, that is for the longer it does not rain, then the probability of occurrence not rain in the next time interval is getting smaller.

INTRODUCTION
Rain is a phenomenon whose occurrence is random, both in space and time. Therefore, the phenomenon of rain can be modeled as a point-process. In the analysis of rainfall phenomena, rainfall intensity at a certain period of time and area plays an important role. In point process models, intensity known as the conditional intensity function or hazard function.

The conditional intensity function is a convenient and intuitive way of specifying how the present depends on the past in an evolutionary point process. Consider the conditional density f* and its corresponding cumulative distribution function F*. Then the conditional intensity function (hazard function) is defined by $\lambda^*(t) = \frac{f^*(t)}{1-F^*(t)}$.

Some previous researchers have been studying the phenomenon of rain, among others are Smith J.A and Karr A.F (1985), Daniel S.W(1988), Rodriguez I et al (2012), and Behrouz (2014), and Sunusi (2015). In their research, Smith develop maximum likelihood procedures for parameter estimation and model selection that have been used to model rainfall occurrences, including Cox process, Renewal processes, etc. Daniel examines Rainfall Intensity using Weibull distribution assumptions in estimating the daily Surface Runoff. A linear point process was studied by Behrouz and then based on introducing conditional intensity function. In this study, the incidence of rainfall is assumed to follow the process of renewal with the inter event time is Weibull and Power Law distributed.

The purpose of this paper is to compare between Weibull and Power Law model in modeling conditional intensity function (hazard function) of rainfall data. For this purpose, the next session will first be described a model of non Homogeneous Poisson Process (NHPP) with intensity function increasing monotone to the inter event time is the Weibull models for $\alpha > 1$ and Power Law models for $\alpha > 1$.

NOTATION AND ASSUMPTION

Intensity Function Of Nhpp
Parameters on non homogeneous Poisson process called as intensity function can be modeled with some models, one of which is the model of Gompertz (G). The function of the intensity of the hazard rates of time between events (Jaroengeratikum et al, 2011). A non homogeneous Poisson process is a stochastic process that can be used in various ways. One is to estimate the chances of a lot of claims that come at a certain time interval. Let $N = \{N(t): t \in [0, T]\}$, where $N(t)$ is number of event in time interval $[0, t), t \geq 0$. Assume that N modeled as non homogeneous Poisson process with the intensity function $\lambda(t|\theta)$ and expected value function $m(t|\theta)$ define as:

IJESMR
International Journal OF Engineering Sciences & Management Research

$$\lambda(t|\theta) = \frac{d}{dt} m(t|\theta) = \frac{d}{dt} E[N(t)] \quad (1)$$

CHARACTERISTIC OF INTENSITY FUNCTION OF WEIBULL AND POWER LAW MODEL

Expectation function $m_G(t|\theta)$ of Weibull model is defined as (Tae-Hyun, 2015):

$$m_W(t|\theta) = \left(\frac{t}{\beta}\right)^{\alpha} \quad (2)$$

Using equation (1), the intensity function $\lambda_W(t|\theta)$ of Gompertz model is define by:

$$\lambda_W(t|\theta) = \left(\frac{\alpha}{\beta}\right)\left(\frac{t}{\beta}\right)^{\alpha-1} \quad (3)$$

where

$\alpha > 0$, is shape parameter; $\beta > 0$, is scale parameter.

Expectation function $m_{PL}(t|\theta)$ of Power Law model is defined as (Rannested, 2007):

$$m_{PL}(t|\theta) = \beta t^{\alpha} \quad (4)$$

Using equation (1), the intensity function $\lambda_G(t|\theta)$ of Power Law model is define by:

$$\lambda_{PL}(t|\theta) = \lambda_{PL}(t) = \alpha\beta t^{\alpha-1} \quad (5)$$

where: $\alpha > 0$, is shape parameter, $\beta > 0$, is scale parameter.

Maximum Likelihood Estimation

Maximum Likelihood Estimation (MLE) method is one method of estimation which maximizes the likelihood function to get the estimated parameters. Suppose a function of the density of the opportunities given by $f(t;\theta)$ with θ is a parameter to be estimated by the maximum likelihood method. Steps for maximum likelihood method:

Make a likelihood function as the following equation:

$$L(\theta|t_1, t_2, \dots, t_n) = \prod_{i=1}^{n} f(t_i;\theta) \quad (6)$$

The *likelihood* function for intensity function of NHPP is:

$$L_{NHPP}(\theta|D) = \left(\prod_{i=1}^{n} \lambda(t_i|\theta)\right) e^{-m(T_n)} \quad (7)$$

with $D = \{n;\ t_1, t_2, \dots, t_n\ ;\ 0 < t_1 < t_2, \dots < t_n\}$

Make a *ln* likelihood function :

$$l = \ln[L] = \ln[L(\theta|D)]$$

The value of the parameter estimation θ is obtained by maximizing the *ln* likelihood function as follows:

$$\frac{\partial l}{\partial \theta} = \frac{\partial(\ln[L])}{\partial \theta} = 0$$

RESULT AND DISCUSSION

Case Study

The data used is data of rainfall in the province (1) South Sulawesi, and (2) Central Sulawesi period January 2005 - March 2016. The station is used for the province of South Sulawesi, namely Paotere station, and for Central Sulawesi namely Bantilan station.

PARAMETER ESTIMATION OF INTENSITY FUNCTION FOR WEIBULL MODEL

Parameters of intensity function for Weibull model with shape parameter α and scale parameter β estimated by MLE. Likelihood function of Weibull model for parameter $\theta(\theta = \alpha, \beta)$ as follows:

$$\begin{aligned} L_W(\theta|D) &= \left(\prod_{i=1}^{n} \lambda_W(t_i;\theta)\right) \exp[-m_W(T_n)] \\ &= \left(\prod_{i=1}^{n} \frac{\alpha}{\beta}\left(\frac{t_i}{\beta}\right)^{\alpha-1}\right) \exp\left[-\left[\frac{T_n}{\beta}\right]^{\alpha}\right] \\ &= \left(\frac{\alpha}{\beta^{\alpha}}\right)^{n} \left(\prod_{i=1}^{n} t_i^{\alpha-1}\right) \exp\left[-\left[\frac{T_n}{\beta}\right]^{\alpha}\right] \end{aligned} \quad (8)$$

IJESMR
International Journal OF Engineering Sciences & Management Research

So that:

$$l = \ln\ [L] = \ln\left[\left(\frac{\alpha}{\beta^{\alpha}}\right)^{n}\left(\prod_{i=1}^{n} t_i^{\alpha-1}\right)\exp\left[-\left[\frac{T_n}{\beta}\right]^{\alpha}\right]\right]$$

$$= n\ln\alpha - n\alpha\ln\beta + (\alpha-1)\sum_{i=1}^{n}\ln t_i - \left[\frac{T_n}{\beta}\right]^{\alpha} \quad (9)$$

Furthermore, to determine the estimated parameter α which causes the function l maximum, then do the following:

$$\frac{\partial l}{\partial\alpha} = -\left(\ln\frac{T_n}{\beta}\right)\left[\frac{T_n}{\beta}\right]^{\alpha} + \frac{n}{\alpha} - n\ln\beta + \sum_{i=1}^{n}\ln t_i = 0 \quad (10)$$

The same is done for the estimator β as follows

$$\frac{\partial l}{\partial\beta} = \frac{\alpha(T_n)^{\alpha+1}}{\beta^{\alpha+1}} - \frac{n\alpha}{\beta} = 0 \quad (11)$$

Based on the equation (10) and (11) does not provide analytical solutions because they depend on the parameters so that the required numerical methods to solve them.

PARAMETER ESTIMATION OF INTENSITY FUNCTION FOR POWER LAW MODEL

Parameters of intensity function for Power Law model with shape parameter α and scale parameter β estimated by MLE. Likelihood functions of Power Law model for parameter $\theta(\theta = \alpha, \beta)$ as follows:

$$L_{PL}(\theta|D) = \left(\prod_{i=1}^{n}\lambda_{PL}(t_i;\theta)\right)\exp[-m_{PL}(T_n)]$$

$$= \left(\prod_{i=1}^{n}\alpha\beta t_i^{\alpha-1}\right)\exp[-\beta\, T_n^{\alpha}]$$

$$= (\alpha\beta)^{n}\left(\prod_{i=1}^{n} t_i^{\alpha-1}\right)\exp[-\beta\, T_n^{\alpha}] \quad (12)$$

So that:

$$l = \ln\ [L] = \ln[(\alpha\beta)^{n}\left(\prod_{i=1}^{n} t_i^{\alpha-1}\right)\exp[-\beta\, T_n^{\alpha}]]$$

$$= n\ln\alpha + n\ln\beta + (\alpha-1)\sum_{i=1}^{n}\ln t_i - \beta\, T_n^{\alpha} \quad (13)$$

Furthermore, to determine the estimated parameter α which causes the function l maximum, then do the following:

$$\frac{\partial l}{\partial\alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n}\ln(t_i) - \beta T_n^{\alpha}\ln T_n = 0 \quad (14)$$

The same is done for the estimator β as follows

$$\frac{\partial l}{\partial\beta} = \frac{n}{\beta} - T_n^{\alpha} = 0 \quad (15)$$

Based on the equation (14) and (15) is not obtained analytic solutions because they depend on the parameters so that the required numerical methods to solve them, so we used the Newton Raphson method. Parameter estimation formula θ in iteration (k + 1)-*th* in the process of iteration (k = 0,1,2, ...). Parameter estimation of the models can be shown in the form:

$$\begin{bmatrix}\hat{\alpha}^{(k+1)}\\ \hat{\beta}^{(k+1)}\end{bmatrix} = \begin{bmatrix}\hat{\alpha}^{(k)}\\ \hat{\beta}^{(k)}\end{bmatrix} - \begin{bmatrix}\dfrac{\partial^2 l^2(\alpha^{(k)})}{\partial^2\alpha^2} & \dfrac{\partial^2 l^2(\alpha^{(k)}\beta^{(k)})}{\partial\alpha\partial\beta}\\ \dfrac{\partial^2 l^2(\alpha^{(k)}\beta^{(k)})}{\partial\alpha\partial\beta} & \dfrac{\partial^2 l^2(\beta^{(k)})}{\partial^2\beta^2}\end{bmatrix}^{-1}\begin{bmatrix}\dfrac{\partial l(\alpha^{(k)})}{\partial\alpha}\\ \dfrac{\partial l(\beta^{(k)})}{\partial\beta}\end{bmatrix}$$

To facilitate the calculation of Newton Raphson iteration process is done with the help of software Rstudio. The results are given in Table 1.

IJESMR
International Journal OF Engineering Sciences & Management Research

Tabel 1. Parameter Estimation of Conditional Intensity of Models

No.	Model	Parameter	
		$\hat{\alpha}$	$\hat{\beta}$
(1)	Weibull	0.7527	0.4359
(2)	Power Law	0.7527	1.8681

Based on Table 1, the obtained form of conditional intensity for Weibull and Power Law model as follows:

$$\lambda_W(t|\theta) = \left(\frac{0.7527}{0.4359}\right)\left(\frac{t}{0.4359}\right)^{0.2473} \quad (16)$$

and

$$\lambda_{PL}(t|\theta) = 0.7527.1.8681\, t^{0.2473}. \quad (17)$$

The following will be given the results of the calculation of the conditional intensity for Weibull and Power Law models are shown in Table 2.

Based on Table 1, the obtained form of conditional intensity for Weibull and Power Law model as follows:

$$\lambda_W(t|\theta) = \left(\frac{0.7527}{0.4359}\right)\left(\frac{t}{0.4359}\right)^{0.2473} \quad (18)$$

and

$$\lambda_{PL}(t|\theta) = 0.7527.1.8681\, t^{0.2473}. \quad (19)$$

The following will be given the results of the calculation of the conditional intensity for Weibull and Power Law models are shown in Table 2.

Table 2. Summary of Conditional Intensity Value of Weibull and Power Law Models

Period	Inter Event time (days)	$\lambda_W(t\|\theta)$	$\lambda_{PL}(t\|\theta)$
[1 Jan , 31 May 2012)	116	0,653711	0,433986
[1 May, 31 Aug 2012)	98	0,681547	0,452466
[31 Aug, 31 Dec 2012)	97	0,683278	0,453615
[31 Des , 2 May 2013)	112	0,659408	0,437769
[2 May, 1 Sept 2013)	75	0,728154	0,483408
[1 Sept, 1 Jan 2014)	111	0,660872	0,438741
[1 Jan , 2 May 2014)	119	0,649596	0,431254
[2 May, 1 Sept 2014)	99	0,679838	0,451331
[1 Sept, 1 Jan 2015)	87	0,701912	0,465986
[1 Jan , 2 May 2015)	122	0,645608	0,428607
[2 May, 1 Sept 2015)	121	0,646924	0,42948
[1 Sept, 1 Jan 2016)	118	0,650953	0,432155
[1 Jan , 1 May 2016)	83	0,71013	0,471442
[1 May, 31 Aug 2016)	88	0,699931	0,464671

Table 2 shows that the highest value of the conditional intensity (0.728154) for both models in the interval of observation is contained in the time interval 2 May until 1 September 2013 with the lowest inter event time (75 days). Instead, the lowest conditional intensity for the two models lies in the time interval Jan 1 - 2 May 2015 with the highest inter event time (122 days). In addition, the results indicate that the value of Conditional Intensity for Weibull and Power Law models for the value of shape parameter $\alpha < 1$ indicate a tendency getting

IJESMR
International Journal OF Engineering Sciences & Management Research

smaller if the time between occurrence increases. This means that the longer the time interval does not rain, then no rain chance on [t, t + 1) is getting smaller.

CONCLUSION

This study shows that the determination of the value of the conditional intensity on a daily rainfall data indicate that the use of Weibull and Power Law models provide the same general characteristics, that is the longer it does not rain, then the probability of occurrence not rain in the next time interval is getting smaller.

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