

A TIME TRUNCATED SPECIAL PURPOSE DOUBLE SAMPLING PLAN DSP(0,1) FOR COMPOUND RAYLEIGH DISTRIBUTION

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ABSTRACT

In this paper, special purpose double sampling plan DSP(0,1) is developed assuming that lifetime of the test units follow Compound Rayleigh distribution and the life test is terminated at a prefixed time. The minimum sample size required for ensuring the specified mean life at specified consumer's confidence level have been determined. The Operating characteristics values for various quality levels are obtained and the results are discussed with the help of tables and examples. The minimum mean ratios are also obtained for a specified level of producer's risk.

KEYWORDS: Truncated life tests, Acceptance sampling plan, Consumer's Risk, Producer's Risk, Compound Rayleigh Distribution

INTRODUCTION

In a competitive economy, goods stand in the market if they are of good quality. A consumer wants products of good quality at reasonable and affordable prices. Here the 'quality' can be defined in two different ways. In one sense, goods are said to be of good quality if they satisfy the consumer. In another sense, goods are said to be of good quality if they meet the expected functional use. Example for second concept, ball-bearings within the specification limits is said to be in 'control production process'. Every unit of production is tested for the standards specified. The units which do not meet the specifications are rejected. The rejected units are said to be of bad quality; as such they are not put to use. Statistical quality control is the procedure for the control of quality by the application of the theory of probability to the results of inspection of samples of the population. Sampling plans are used in the area of quality and reliability analysis. When the quality of product is related to its lifetime, it is called as life test.

In most of the life testing sampling plans a common constraint is the duration of the total time spent on the test. It is usual to terminate a life test by prefixed time and record the number of failures till that time .If the number of observed failures at the end of the fixed time is not greater than the specified acceptance number, then the lot will be accepted. The test may get terminated before the pre specified time is reached when the number of failures exceeds the acceptance number in which case the decision is to reject the lot. Two risks are continually associated to a time truncated acceptance sampling plan. The probability of accepting a bad lot is known as the consumer's risk and the probability of rejecting a good lot is called the producer's risk. For such a truncated life test and the associated decision rule we are interested in obtaining the smallest sample size to arrive at a decision where the life time of an item follows Compound Rayleigh distribution.

The priority of every sampling plan is the reduction of cost and time, which depends on the sample size. In a single sampling plan, a decision regarding rejection and acceptance of the lot is taken on the basis of single sample. From Cameron table (1952), one can observe a jump between the operating ratios of single sampling plan with c=0 and c=1 and slow reduction of operating ratios for other values of c. It may also be seen that, in between the operating characteristic (OC) curves of single sampling plan with c=0 and c=1 plans, there is a vast gap to be filled which leads one to assess the possibility of designing plans having OC curves lying between the OC curves of c=0 and c=1 plan. To overcome such situation Craig (1981) have proposed Double sampling plan with acceptance numbers 0 and 1 and rejection number 2. Vijayaragavan (1990), has presented tables for the selection of DSP (0,1) plan for attributes under Poisson and Binomial conditions of sampling. Dodge and Romig(1959) have studied the use of DSP (0,1) plan to product characteristics involving costly and destructive testing. SudamaniRamaswamy and Sutharani., (2014), discussed the special purpose double sampling plan of type DSP (0,1) for truncated life test using minimum angle method. SudamaniRamaswamy



and Jaishree (2014) proposed a new approach of designing special purpose double sampling plan of type DSP (0,1) for truncated life test assuming that the experiment is truncated at pre-assigned time, when the lifetime of the items follows different distributions. In this paper, designing double sampling plan of type DSP(0,1) for truncated life test is proposed, assuming that the experiment is truncated at pre-assigned time when the lifetime of the items follow Compound Rayleigh distribution.

The minimum sample size required for ensuring the specified mean life at specified consumer's confidence level has been determined. The Operating characteristics values for various quality levels are obtained and the results are discussed with the help of tables and examples. The minimum mean ratios are also obtained for a specified level of producer's risk.

COMPOUND RAYLEIGH DISTRIBUTION

The Rayleigh distribution played an important role in modelling the lifetime of random phenomenon. It arises in many areas of applications, including reliability, life testing and survival analysis. Bhupendra Singh, K.K. Sharma and DushyantTyagi (2013) have developed a reliability single sampling plan assuming that lifetime of the test units follow Compound Rayleigh distribution and the life test is terminated at a prefixed time. This type of sampling plan is used to save the test time in practical situations.

Let X denotes a random variable arising from a Rayleigh distribution with p.d.f.

$$f(t;\theta) = 2\theta t e^{-\theta t}$$
where t > 0 is the lifetime, and $\theta > 0.$
(1)

The corresponding hazard function is

0.2

 $h(t) = 2\theta t, t > 0$

The mean survival time and the cumulative distribution function of the Rayleigh model are given by

$$E(t) = \frac{1}{2} \sqrt{\frac{\pi}{\theta}}$$
⁽²⁾

$$F(t) = 1 - e^{\frac{-t}{\theta}} \tag{3}$$

In life testing experiments, it is expected that the environmental conditions can not be remained same during the testing time. Therefore, it seems logical to treat the parameters involved in the life time model as random variables. In view of this, if the parameter θ is itself a random variable, then the distribution of lifetime of each item is a Compound Rayleigh distribution. The particular form of θ , which is considered here, is the gamma p.d.f.

The parameters B and δ are scale and shape parameters, respectively. The resulting Compound distribution has p.d.f.

$$f(t,\alpha,B) = \int_{0}^{\infty} 2\theta t \, e^{-\theta t^{2}} \, \frac{B^{\delta} \theta^{\delta-1} e^{-B\theta}}{\Gamma \delta} \, d\theta$$
$$= 2\delta B^{\delta} t \left(B + t^{2}\right)^{-(\delta+1)} \tag{5}$$

The mean survival time and the cumulative distribution function of the Compound Rayleigh model are given by

$$\mu = E(t) = \frac{\sqrt{B\pi}\Gamma\left(\delta - \frac{1}{2}\right)}{2\Gamma\delta}$$
(6)

and

$$F(t,B,\delta) = 1 - B^{\delta} (B + t^2)^{-\delta}, \qquad t > 0$$
(7)

OPERATING PROCEDURE OF SPECIAL PURPOSE DOUBLE SAMPLING PLAN OF TYPE DSP (0,1)

According to Hald (1981), the operating procedure of DSP (0,1) is as follows;

(i) From a lot, select a samle of size n_1 , and observe the number of defectives, d_1 .



- (ii) If $d_1=0$, accept the lot; If $d_1 > 1$, reject the lot;
- (iii) If $d_1=1$, select a second sample of size n_2 and observe d_2 .
- (iv) If $d_2=0$, accept the lot, otherwise reject the lot.

OPERATING PROCEDURE FOR DSP (0,1) SAMPLING PLAN FOR TRUNCATED LIFE TEST

- (i) From a lot, select a sample of size n_1 , and observe the number of defectives d_1 , during the time t_0 .
- (ii) If $d_1=0$, accept the lot; If $d_1>1$, reject the lot;
- (iii) If $d_1=1$, select a second sample of size n_2 and observe d_2 , during the time t_0 .
- (iv) If $d_2=0$, accept the lot, otherwise reject the lot.

The following is the operating procedure for special purpose double sampling plan for life test in the form of a flow chart.

FLOW CHART



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NOTATIONS

- n₁ Size of the first sample
- n₂ Size of the second sample
- d₁ Defectives in the first sample
- d_2 Defectives in the second sample
- α Producer's risk
- β Consumer's risk
- t Termination time
- δ Shape parameter
- B Scale parameter
- p Probability of failure before time t
- pa Probability of acceptance of lot
- μ_0 Specified mean life

DESIGN OF THE SAMPLING PLAN

The main objective of this plan is to set a lower confidence limit P^* , on the products mean lifetime μ and to test whether the lifetime of the product is longer than our expectation. It is assumed that the lot size is large enough to use binomial distribution to find the probability of acceptance. The probability of acceptance L(p) for this sampling plan is calculated using the following equation.

$$L(p) = (1-p)^{n_1} + n_1 p (1-p)^{n_1+n_2-1}$$
(8)

where $n_1 = n$, $n_2 = kn$ and p is the failure probability.

The required sample size n is the smallest positive integer that satisfies the following inequality.

$$L(p) = (1 - p_0)^{n_1} + n_1 p_0 (1 - p_0)^{n_1 + n_2 - 1} \le \beta$$
(9)

According to Bhupendra Singh et.al, (2013) the value of p₀ is given for Compound Rayleigh distribution as

 $p_0 = 1 - B^{\delta} (B + t_0^2)^{-\delta}$

$$=1 - \frac{1}{(1 + \frac{t_0^2}{B})^{\delta}}$$
(10)

where
$$B = \left(\frac{2\mu\Gamma\delta}{\sqrt{\pi}\Gamma\left(\delta - \frac{1}{2}\right)}\right)^{\frac{1}{2}}$$
 (11)

Substituting the value of B and $t = a\mu_0$, we get

$$p_{0} = 1 - \frac{1}{1 + \left(\frac{a\sqrt{\pi}\Gamma\left(\delta - \frac{1}{2}\right)}{2\frac{\mu}{\mu_{0}}\Gamma\delta}\right)^{2}}$$
(12)



The minimum values of $n_1 = n$ satisfying equation (9) are obtained and given in Table 1 for various values of β and t/ μ_0 . The shape parameter is fixed as 1, and if some other parameters are involved then they are assumed to be known.

By fixing the time termination ratio t/μ_0 as 0.628, 0.912, 1.257, 1.571, 2.356, 3.141 and 4.712, the consumer's risk β as 0.75, 0.90, 0.95, 0.99 and the mean ratio $\mu/\mu_0 = 2, 4, 6, 8, 10, 12$, we can find the size of the first and the second samples n_1 and n_2 by substituting the failure probability p in the equation (8) and satisfying the following inequality at worst case $(\mu = \mu_0)$. (13)

$$L(p) \leq \beta$$

The sample sizes of the first sample, n₁ are calculated for the Compound Rayleigh distribution and is presented in Table 1.

OPERATING CHARACTERISTICS (OC) CURVE

The OC function of the sampling plan is the probability of accepting a lot and is given by

$$L(p) = (1-p)^{n_1} + n_1 p (1-p)^{n_1+n_2-1}$$
(14)

where $p = F(t, B, \delta)$ is treated as a function of lot quality. The OC values for different combinations of the values of confidence level are computed and presented in Table 2.

For a given value of the producer's risk α , the minimum value of μ/μ_0 is determined such that it satisfies the following inequality.

$$(1-p)^{n_1} + n_1 p (1-p)^{n_1+n_2-1} \ge 1-\alpha$$
(15)

and are presented in Table 3.

Figure 1 OC values vs. mean ratio μ/μ_0 with experiment time ratio a = 0.628



EXAMPLE

Assume that an experimenter wants to establish that the lifetime of the electrical devices produced in the factory ensures that the true unknown mean life is at least 1000 hours with consumer's risk $\beta = 0.10$. It is desired to stop the experiment at t = 628 hours. It is assumed that k = 0.5. Based on consumer's risk values and the test termination time, the minimum sample size is determined using the special purpose double sampling plan of type DSP(0,1) for truncated life test. Let the distribution followed be Compound Rayleigh distribution, then we



get the sample size as 5 from Table 1. The lot is accepted, at a given mean ratio $\mu/\mu_0 = 2$, during 628 hours, with the plan parameters $(n_1,n_2) = (5, 2.5) \cong (5, 3)$ satisfying the consumer's risk. From the Table 2, one can observe that the probability of acceptance for this sampling, when $\mu/\mu_0 = 2$ is 0.574666. For the same measurements and plan parameters the probability of acceptance is 0.999012, when the ratio of unknown average life is 12. For the same conditions, when the time of experiment is 4712 hours, the probability of acceptance for ratio $\mu/\mu_0 = 2$ is 0.013285, with the plan parameters $(n_1, n_2) = (2, 1)$. From Table 3, at a given mean ratio $\mu/\mu_0 = 2$ during 628 hours, the minimum ratio of μ to specified μ_0 for the acceptability of a lot with producer's risk 0.05 is given as 4.224.

CONCLUSION

It is observed from Figure 1 and from Table 3 that the Operating Characteristic values of Compound Rayleigh distribution increases and it is nearest to unity when μ/μ_0 increases. When there is an increase in confidence level, the minimum ratio and the sample size are also increases. For various experiment time ratio, the minimum sample size required to make a decision increases with an increases in the confidence level. This sampling plan can be suggested for the industrial purposes to save time and cost of the life test experiments.

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β	k		t/μ_0								
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712		
	0	5	3	3	3	2	2	2	2		
	0.5	4	2	2	2	2	1	1	1		
	1	3	2	2	2	1	1	1	1		
	1.5	3	2	2	1	1	1	1	1		
	2	3	2	1	1	1	1	1	1		
0.25	2.5	3	2	1	1	1	1	1	1		
	3	3	2	1	1	1	1	1	1		
	3.5	3	2	1	1	1	1	1	1		

Table 1 : Minimum Sample size (n_1) for DSP(0,1) plan when the life time of the items follows Compound Rayleigh distribution with $\delta = 1$



	4	3	2	1	1	1	1	1	1
	4.5	3	2	1	1	1	1	1	1
	5	3	2	1	1	1	1	1	1
	5.5	3	2	1	1	1	1	1	1
	6	3	2	1	1	1	1	1	1
	6.5	3	2	1	1	1	1	1	1
	7	3	2	1	1	1	1	1	1
	7.5	3	2	1	1	1	1	1	1
	8	3	2	1	1	1	1	1	1
	8.5	3	2	1	1	1	1	1	1
	9	3	2	1	1	1	1	1	1
	9.5	3	2	1	1	1	1	1	1
	10	3	2	1	1	1	1	1	1
	0	7	4	4	3	3	2	2	2
	0.5	5	3	3	2	2	2	2	2
	1	4	3	2	2	2	1	1	1
	1.5	4	3	2	2	1	1	1	1
	2	4	3	2	2	1	1	1	1
	2.5	4	3	2	2	1	1	1	1
	3	4	3	2	2	1	1	1	1
	3.5	4	3	2	2	1	1	1	1
	4	4	3	2	2	1	1	1	1
	4.5	4	3	2	2	1	1	1	1
0.10	5	4	3	2	2	1	1	1	1
0.10	5.5	4	3	2	2	1	1	1	1
	6	4	3	2	2	1	1	1	1
	6.5	4	3	2	2	1	1	1	1
	7	4	3	2	2	1	1	1	1
	7.5	4	3	2	2	1	1	1	1
	8	4	3	2	2	1	1	1	1
	8.5	4	3	2	2	1	1	1	1
	9	4	3	2	2	1	1	1	1
	9.5	4	3	2	2	1	1	1	1
	10	4	3	2	2	1	1	1	1
	0	8	5	4	4	3	3	3	2
	0.5	6	4	3	3	2	2	2	2
	1	5	3	3	2	2	2	2	1



0.05	1.5	5	3	2	2	2	2	1	1
	2	5	3	2	2	2	1	1	1
	2.5	5	3	2	2	2	1	1	1
	3	5	3	2	2	2	1	1	1
	3.5	5	3	2	2	2	1	1	1
	4	5	3	2	2	2	1	1	1
	4.5	5	3	2	2	2	1	1	1
	5	5	3	2	2	2	1	1	1
	5.5	5	3	2	2	2	1	1	1
	6	5	3	2	2	2	1	1	1
	6.5	5	3	2	2	2	1	1	1
0.05	7	5	3	2	2	2	1	1	1
	7.5	5	3	2	2	2	1	1	1
	8	5	3	2	2	2	1	1	1
	8.5	5	3	2	2	2	1	1	1
	9	5	3	2	2	2	1	1	1
	9.5	5	3	2	2	2	1	1	1
	10	5	3	2	2	2	1	1	1
	0	11	7	5	5	4	3	3	3
	0.5	8	5	5	4	3	2	2	2
	1	7	5	3	3	2	2	2	2
	1.5	7	4	3	3	2	2	2	2
	2	7	4	3	3	2	2	2	2
	2.5	7	4	3	3	2	2	2	2
	3	7	4	3	3	2	2	2	2
	3.5	7	4	3	3	2	2	2	2
	4	7	4	3	3	2	2	2	2
	4.5	7	4	3	3	2	2	2	2
0.01	5	7	4	3	3	2	2	2	2
0.01	5.5	7	4	3	3	2	2	2	2
	6	7	4	3	3	2	2	2	2
	6.5	7	4	3	3	2	2	2	2
	7	7	4	3	3	2	2	2	2
	7.5	7	4	3	3	2	2	2	2
	8	7	4	3	3	2	2	2	2
	8.5	7	4	3	3	2	2	2	2
	9	7	4	3	3	2	2	2	2



9.5	7	4	3	3	2	2	2	2
10	7	4	3	3	2	2	2	2



Table 2 : Probability of acceptance for DSP	o (0,1) plan with	h k = 0.5, when th	e lifetime of the	units follows
Compound Rayleigh distribution				

β	n	t/µ0			μ/	μ/μο			
			2	4	6	8	10	12	
	4	0.628	0.682395	0.960428	0.990957	0.996985	0.998734	0.999381	
	2	0.942	0.713446	0.960089	0.990534	0.996796	0.998645	0.999335	
	2	1.257	0.510006	0.900034	0.973269	0.990506	0.99589	0.997956	
0.25	2	1.571	0.347239	0.814149	0.943351	0.978704	0.990511	0.995204	
	2	2.356	0.130372	0.558451	0.81425	0.917864	0.960032	0.978719	
	1	3.141	0.463998	0.776803	0.9083	0.959046	0.979827	0.989156	
	1	3.927	0.374395	0.679271	0.844602	0.922521	0.959078	0.976916	
	1	4.712	0.311316	0.594248	0.77676	0.877478	0.930647	0.959032	
	5	0.628	0.574666	0.93978	0.985828	0.995223	0.997984	0.999012	
	3	0.942	0.500602	0.911264	0.977603	0.992239	0.996681	0.998361	
	3	1.257	0.267096	0.794142	0.939214	0.977539	0.990086	0.995017	
0.10	2	1.571	0.347238	0.814149	0.943351	0.978704	0.990511	0.995204	
	2	2.356	0.130371	0.558452	0.81425	0.917864	0.960032	0.97872	
	2	3.141	0.054219	0.347457	0.643433	0.814301	0.900176	0.94341	
	2	3.927	0.025469	0.211267	0.479616	0.686932	0.814209	0.887371	
	2	4.712	0.013285	0.130371	0.347384	0.558452	0.713181	0.81425	
	6	0.628	0.476904	0.916337	0.979728	0.993092	0.997071	0.99856	
	4	0.942	0.334368	0.852465	0.960428	0.985964	0.993931	0.996985	
	3	1.257	0.267096	0.794142	0.939214	0.977539	0.990086	0.995017	
0.05	3	1.571	0.133584	0.647868	0.877088	0.951051	0.977552	0.988467	
	2	2.356	0.130371	0.558452	0.81425	0.917864	0.960032	0.97872	
	2	3.141	0.054219	0.347457	0.643433	0.814301	0.900176	0.94341	
	2	3.927	0.025469	0.211267	0.479616	0.686932	0.814209	0.887371	
	2	4.712	0.013285	0.130371	0.347384	0.558452	0.713181	0.81425	
	8	0.628	0.318329	0.863333	0.964934	0.987794	0.994771	0.997415	
	5	0.942	0.216687	0.788397	0.93978	0.978143	0.990443	0.995223	
	5	1.257	0.063096	0.573965	0.848675	0.939618	0.972317	0.985786	
0.01	4	1.571	0.048468	0.495548	0.800389	0.915941	0.96034	0.979292	
	3	2.356	0.024358	0.315659	0.648029	0.827415	0.911145	0.951086	
	2	3.141	0.054219	0.347457	0.643433	0.814301	0.900176	0.94341	
	2	3.927	0.025469	0.211267	0.479616	0.686932	0.814209	0.887371	
	2	4.712	0.013285	0.130371	0.347384	0.558452	0.713181	0.81425	



Table 3 : Minimum ratio of true value μ to specified μ_0 for the acceptability of a lot with producer's risk 0.05

β	k	t/µ0									
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712		
	0	3.428	3.739	4.988	6.234	6.893	9.189	11.489	13.785		
	0.5	3.739	3.739	4.988	6.234	9.349	7.482	9.354	11.223		
	1	3.585	4.277	5.707	7.133	6.893	9.189	11.489	13.785		
	1.5	3.880	4.660	6.218	5.140	7.709	10.278	12.849	15.418		
0.25	2	4.115	4.959	4.439	5.548	8.319	11.092	13.866	16.638		
	2.5	4.310	5.207	4.701	5.875	8.810	11.746	14.685	17.620		
	3	4.478	5.419	4.921	6.150	9.222	12.295	15.372	18.445		
	3.5	4.625	5.604	5.111	6.387	9.579	12.770	15.965	19.157		
	4	4.757	5.769	5.278	6.596	9.892	13.188	16.488	19.784		
	4.5	4.875	5.917	5.428	6.783	10.173	13.562	16.955	20.345		
	5	4.983	6.052	5.563	6.952	10.426	13.900	17.378	20.852		
	0	4.153	4.497	6.001	6.234	9.349	9.189	11.489	13.785		
	0.5	4.224	4.767	6.361	6.234	9.349	12.464	15.583	18.698		
	1	4.191	5.377	5.707	7.133	10.696	9.189	11.489	13.785		
	1.5	4.523	5.820	6.217	7.771	7.709	10.278	12.849	15.418		
0.10	2	4.789	6.172	6.618	8.271	8.319	11.091	13.866	16.638		
	2.5	5.011	6.465	6.948	8.684	8.810	11.746	14.685	17.620		
	3	5.201	6.717	7.231	9.037	9.223	12.295	15.372	18.445		
	3.5	5.369	6.938	7.478	9.346	9.579	12.770	15.965	19.157		
	4	5.519	7.135	7.698	9.621	9.892	13.188	16.488	19.784		
	4.5	5.654	7.312	7.896	9.868	10.173	13.562	16.955	20.345		
	5	5.778	7.474	8.075	10.093	10.426	13.900	17.378	20.852		
	0	4.471	5.141	6.001	7.500	9.349	12.464	15.583	13.785		
	0.5	4.660	5.608	6.361	7.950	9.349	12.464	15.583	18.698		
	1	4.719	5.377	7.175	7.133	10.696	14.260	17.828	13.785		
	1.5	5.085	5.820	6.217	7.771	11.653	15.537	12.849	15.418		
0.05	2	5.379	6.172	6.618	8.271	12.403	11.091	13.866	16.638		
	2.5	5.624	6.465	6.948	8.684	13.023	11.746	14.685	17.620		
	3	5.836	6.717	7.231	9.037	13.553	12.295	15.372	18.445		
	3.5	6.022	6.938	7.478	9.346	14.016	12.770	15.965	19.157		
	4	6.188	7.135	7.698	9.621	14.428	13.188	16.488	19.784		
	4.5	6.338	7.312	7.896	9.868	14.799	13.562	16.955	20.345		
	5	6.475	7.474	8.075	10.093	15.135	13.900	17.378	20.852		



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0.01	0	5.511	0.227	0.000	0.574	11.47/	12.404	15.505	10.070
	0.5	5.426	6.336	8.455	9.352	11.923	12.464	15.583	18.698
	1	5.629	7.078	7.175	8.967	10.696	14.260	17.828	21.392
	1.5	6.055	6.785	7.767	9.706	11.653	15.536	19.423	23.306
	2	6.397	7.183	8.236	10.293	12.403	16.536	20.673	24.806
	2.5	6.684	7.516	8.627	10.782	13.023	17.362	21.707	26.046
	3	6.932	7.802	8.963	11.202	13.553	18.068	22.589	27.105
	3.5	7.150	8.054	9.258	11.570	14.016	18.686	23.361	28.031
	4	7.346	8.278	9.521	11.899	14.428	19.235	24.048	28.855
	4.5	7.522	8.481	9.758	12.195	14.798	19.729	24.665	29.596
	5	7.683	8.666	9.974	12.465	15.135	20.178	25.227	30.270