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RESOURCE CONSTRAINED PROJECT SCHEDULING USING AHP WITH TOPSIS

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ABSTRACT

Quite a few approaches are followed in project scheduling under multiple resources. Typically, priority for each activity is obtained using qualitative data with AHP. In this paper, both quantitative and qualitative data are considered in a fuzzy environment. This paper develops an evaluation model based on the Analytical Hierarchy Process (AHP) and the Technique for Order Performance by Similarity to Ideal Solution (TOPSIS). The AHP is employed to analyze the structure of the project and to determine the weights for the constraints, and TOPSIS method is used to develop the weights for the resources consumed by the activities. A weighted sum of resources for each activity is obtained using these weights of the resources and then ranking the activities by considering the weighted sum of the activities. Scheduling the activities is carried out taking into consideration the rank of the activity as well as the precedence relationship and resource requirements and the final project schedule is obtained. The method is demonstrated through numerical illustration.

INTRODUCTION

Project scheduling is the discipline for stating how to complete a project within a certain timeframe, usually with defined stages and with designated resources. Project scheduling involves the development of a project base plan which specifies for each activity the precedence and resource feasible start and completion dates, the amount of the various resource types that will be needed during each time period and as a result the corresponding budget required for the execution of the project (Brucker et al. (1999)).

Resource allocation is used to assign the available resources in an economic way. It is part of resource management. Project management is considered to be an important field in production mainly because many of the industrial activities can also be viewed as project management problems. In project management, resource allocation is the scheduling of activities and the resources required by those activities while taking into consideration both the resource availability and the project time [12].

The AHP has been proposed in recent literature as an emerging solution approach to large, dynamic and complex real world multi-criteria decision-making problems (Stan Lipovetsky (1996)). Jiaqin Yang and Ping Shi (2002) proposed the AHP for evaluating firm's overall performance, especially for firms under its unique economy, financial and marketing conditions in China. Behzadian et al. (2012) asserted a TOPSIS based model for multi criteria decision making in another study. Onder and Dag (2013) proposed an approach based on AHP and improved TOPSIS for the supplier selection problem. Zaidan et al. (2015) presented an approach based on integrated AHP and TOPSIS to select the optimal open-source EMR software packages.

METHODS

2.1 The AHP Method

The analytical hierarchy process (AHP) is a decision aiding technique which aims at quantifying relative priorities for a specified set of alternatives on a ratio scale. It's a powerful and flexible decision making process to facilitate people set priorities and make the best decision when both qualitative and quantitative aspects of a decision need to be considered (Satty (1994)). In the early 1970's Satty developed AHP which is a problem solving framework based on the inherent human ability to make sound judgment for small problems. A hierarchy of the problem is structured to encompass the basic elements. The objective is to derive priorities on the elements in the last level that best reflect their relative impact on the focus of the hierarchy. To apply the principle of comparative judgments, a matrix is set up to carry out pair wise comparisons of relative importance of the elements in the second level with respect to the overall focus of the first level. AHP uses pair wise comparison to deliver ratio-based priorities (Flavio et al. (2003)). AHP is an emerging solution approach to large, dynamic and complex real world Multi Criteria Decision Making (MCDM) problems (Stan Lipovetsky (1996)).

In the first step, the project scheduling problem is structured as a hierarchy. AHP initially breaks down a complex multi criteria decision making problem into a hierarchy of consistent decision elements. A hierarchy



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has at least three levels: overall goal of the problem at the top, multiple criteria that describe the alternatives in the middle, and decision alternatives at the bottom (Albayrak & Erensal, 2004).

The second step is the comparison of the alternatives and the criteria. Once the hierarchy is constructed, prioritization procedure starts in order to determine the relative importance of the criteria within each level. A pairwise comparison starts from the second level and finishes in the lowest level, alternatives. In AHP, multiple pairwise comparisons are based on a standardized comparison scale of nine levels (Table1).

Table 1: Pair-wise comparison scale for AHP preferences Satty T.L. (1980) Satty T.L. and Kearns KP. (1991)

Numerical Rating	Verbal judgments of preferences
9	Extremely preferred
8	Very strongly to extremely
7	Very strongly preferred
6	Strongly to very strongly
5	Strongly preferred
4	Moderately to strongly
3	Moderately preferred
2	Equally to moderate
1	Equally preferred

Let C_1, C_2, \dots, C_n be the set of criteria. The pair wise comparison on the criteria, C_i and C_j are represented by $n \times n$ matrix.

$A = [a_{ij}]$, where $i, j = 1, 2, 3, \dots, n$

The entries a_{ij} are defined by the external knowledge base or by the customer preference. If $a_{ij} = \alpha$, then $a_{ji} = 1/\alpha$, $\alpha \neq 0$.

$$A = \begin{pmatrix} 1 & a_{12} & \dots & a_{1n} \\ \frac{1}{a_{12}} & 1 & \dots & a_{2n} \\ a_{12} & & & \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & \vdots & 1 \\ a_{1n} & a_{2n} & & \end{pmatrix} \tag{1}$$

After the formulation of the pairwise comparison matrix, the mathematical process commences to normalize and find the relative weights for each matrix. The relative weights are given by the right eigenvector (w) corresponding to the largest eigenvalue (λ_{max}), as

$$A_w = \lambda_{max} w \tag{2}$$

If $\vec{Aw} = \lambda \vec{w}$, as demonstrated above then \vec{w} is an Eigen vector of matrix A, and the Eigen value is $\lambda = n$, however the results are rarely consistent. The AHP methodology calculates a consistency Index (CI) as follows:

$$CI = \frac{\lambda_{max} - n}{n - 1} \tag{3}$$

Modify the comparison matrix, so that judgments should be reviewed and improved, until C.R is less than 0.1 (depending on size of matrix). Finally, the consistency ratio (CR) is calculated by comparing the consistency index to the Random consistency.

$$CR = \frac{CI}{RI} \tag{4}$$

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Where RI is the random index for matrices with random generated pairwise comparisons, the Table 4.3 with RI values computed by simulation is used for calculation of the CR ratio. A CR value, less than 0.1 is considered to be with reasonable consistency (Satty, 1994).

Table 2 Random consistency index for various matrix sizes (Satty, 1994)

N	1	2	3	4	5	6	7	8	9	10	11	12	13
R.I	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49	1.51	1.54	1.56

2.2 The TOPSIS method

The TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) was first developed by Hwang & Yoon (1981). It consists of both positive-ideal solution and negative-ideal solution. The positive-ideal solution is the one that maximizes the benefit criteria and minimizes the cost criteria, where as the negative-ideal solution maximizes the cost criteria and minimizes the benefit criteria (Wang & Elhag, 2006). According to this technique, the best alternative would be the one that is nearest to the positive-ideal solution and farthest from the negative-ideal solution (Ertugrul & Karakasoglu, 2007). As the literature says TOPSIS is an effective tool applied to obtain solutions for the MCDM problems (Lai, Liu, & Hwang, 1994; Chen, 2000; Chu, 2002; Chu & Lin, 2002; Wang, Liu, & Zhang, 2005). The TOPSIS method consists of the following steps (Shyur & Shih, 2006):

Step 1:

Establish a decision matrix for the ranking. The structure of the matrix can be expressed as follows

$$D = \begin{matrix} & F_1 & F_2 & \dots & F_j & \dots & F_n \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_i \\ \vdots \\ A_j \end{matrix} & \begin{bmatrix} f_{11} & f_{12} & \dots & f_{1j} & \dots & f_{1n} \\ f_{21} & f_{22} & \dots & f_{2j} & \dots & f_{2n} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ f_{i1} & f_{i2} & \dots & f_{ij} & \dots & f_{in} \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ f_{j1} & f_{j2} & \dots & f_{jj} & \dots & f_{jn} \end{bmatrix} \end{matrix}$$

Where A_j denotes the alternatives j , $j = 1, 2, \dots, J$; F_i represents i th attribute or criterion, $i = 1, 2, \dots, n$, related to i th alternative; and f_{ij} is a crisp value indicating the performance rating of each alternative A_i with respect to each criterion F_j .

Step2: Calculate the normalized decision matrix $R(=[r_{ij}])$. The normalized value r_{ij} is calculated as :

$$r_{ij} = \frac{f_{ij}}{\sqrt{\sum_{j=1}^J f_{ij}^2}} \quad j = 1, 2, \dots, J; i = 1, 2, \dots, n.$$

Step 3: Calculate the weighted normalized decision matrix by multiplying the normalized decision matrix by its associated weights. The weighted normalized value v_{ij} is calculated as:

$$V_{ij} = w_i \times r_{ij}, \quad j = 1, 2, \dots, J; i = 1, 2, \dots, n.$$

Where w_i represents the weight of the i th attribute or criterion

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Step4: Determine the positive-ideal and negative-ideal solutions

$$A^* = \{v_1^*, v_2^*, \dots, v_i^*\}$$

$$= \left\{ \left(\max_j v_{ij} \mid i \in I' \right), \left(\min_j v_{ij} \mid i \in I'' \right) \right\},$$

$$A^- = \{v_1^-, v_2^-, \dots, v_i^-\}$$

$$= \left\{ \left(\min_j v_{ij} \mid i \in I' \right), \left(\max_j v_{ij} \mid i \in I'' \right) \right\},$$

Where I' is associated with the benefit criteria, and I'' is associated with the cost criteria.

Step5: Calculate the separation measures, using the n-dimensional Euclidean distance. The separation of each alternative from the positive -ideal solution (D_j^*) is given as

$$D_j^* = \sqrt{\sum_{i=1}^n (v_{ij} - v_i^*)^2} \quad j=1,2,\dots,J.$$

Similarly, the separation of each alternative from the negative ideal solution (D_j^-) is as follows:

$$D_j^- = \sqrt{\sum_{i=1}^n (v_{ij} - v_i^-)^2} \quad j=1, 2, \dots, J.$$

Step 6: Calculate the relative closeness to the ideal solution and rank the performance order. The relative closeness of the alternative A_j can be expressed as

$$CC_j^* = \frac{D_j^-}{D_j^* + D_j^-}, j=1,2,\dots,J.$$

Where the CC_j^* index value lies between 0 and 1. The larger the index value means the better the performance of the alternatives.

ILLUSTRATIVE EXAMPLE

An illustrative example is taken to explain the scheduling of a project by using the methodologies AHP and TOPSIS.

Table 3

Activity	Duration (days)	Predecessors	Resource Requirements per day					
			R1	R2	R3	R4	R5	R6
A	6	-	5	2	2	2	7	4
B	3	-	3	5	2	3	9	6
C	4	A	2	4	4	2	3	1
D	6	-	5	4	3	5	5	4
E	7	A,B	3	5	2	3	8	0

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F	5	C	4	1	4	9	2	5
G	2	D	4	1	4	3	9	8
H	2	A,B	5	5	4	0	9	1
I	2	G,H	3	2	4	3	4	2
J	6	F	1	5	4	6	7	3
K	1	C,E	3	3	2	4	5	1
L	2	E,G,H	3	2	2	8	3	4
M	4	I,K	2	2	2	2	4	8
N	2	F,L	1	4	4	3	4	1
O	3	L	5	5	4	6	2	3
P	5	J,M,N	3	2	3	4	7	8
Q	8	O	4	5	4	2	3	4
R	2	D,O	5	3	3	3	7	8
S	6	P,R	2	4	6	2	3	4
T	2	Q	1	6	2	7	5	2
Daily Resource Limit			7	10	10	16	18	13

Source: (Tarek Hegazy, (1999))

Table 3 gives the information about the activities, their duration, precedence relationship of each activity and the resources required for each activity. The illustrative example consists of 20 activities; six resources are involved for each activity to complete and give information about the daily resource limit.

Factors	Availability of resources	Criticality of the resource	Relative cost of resource		
Availability of resources	1	2	2		
Criticality of resources	1/2	1	2		
Relative cost of resources	1/2	1/2	1		
Sum	2	3.5	5		

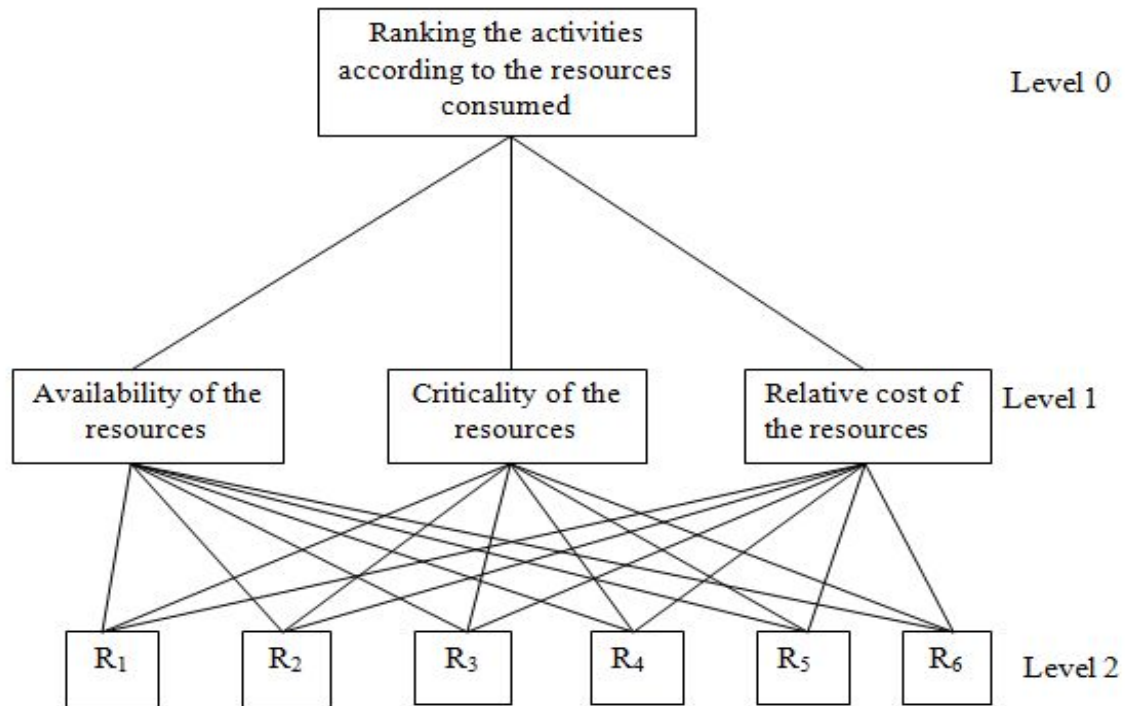


Figure 1 Hierarchical tree of the AHP for the illustrative example

3.1. Calculating the weights of criteria

After forming the hierarchy of the problem, the weights of the criteria are calculated by using AHP method.

Solution Procedure:

- Step1: developing the pair-wise comparison matrix for the resources required for the activities.
- Step2: Developing the normalized comparison matrix for the pair-wisecomparison matrix by dividing the each element in the column by the sum of that particular column.
- Step3: Establishing priority vector
- Step4: Comparison of alternatives
- Step5: Calculating priority vector for alternatives

Table 4 calculating the weightages for the criteria

Normalized matrix					
				Sum	Priority vector
Availability of resources	0.5	0.5714	0.4	1.4714	0.4934
Criticality of resources	0.25	0.2857	0.4	0.9357	0.3198
Relative cost of resources	0.25	0.1428	0.2	0.5928	0.1958
Lambda max	3.0536				
Consistency Index (CI)	0.0268	n=3			
Consistency ratio (CR)	0.0462				

The pairwise comparison matrix and the results obtained from the computations based on pairwise comparison matrix are presented in Table 4. The consistency ratio of the pairwise comparison matrix is calculated as 0.0462 < 0.1. So the weights are shown to be consistent and they are used in the selection process.



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3.2 Calculating the weights of the alternatives

The TOPSIS methodology is used to obtain the weights for the alternatives. At this stage of the decision procedure, establishment of decision matrix is carried out by comparing the alternatives under each of the criteria separately.

Step 1: Establish a decision matrix for the ranking.

Step2: Calculate the normalized decision matrix $R(=[r_{ij}])$.

Step 3: Calculate the weighted normalized decision matrix by multiplying the normalized decision matrix by its associated weights.

Step4: Determine the positive-ideal and negative-ideal solutions

Step5: Calculate the separation measures, using the n-dimensional Euclidean distance.

Step 6: Calculate the relative closeness to the ideal solution and rank the performance order.

Table 5 Evaluation matrix for the alternatives

	C ₁	C ₂	C ₃
Weights	0.4934	0.3198	0.1958
R ₁	3	3	3
R ₂	3	1	3
R ₃	5	5	3
R ₄	5	5	3
R ₅	7	7	7
R ₆	5	3	3
Squared sum	142	94	94

Table 6 Normalized and weighted normalized evaluation of the alternatives

Normalized Decision matrix			
R ₁	0.2518	0.3094	0.3094
R ₂	0.2518	0.1031	0.3094
R ₃	0.4196	0.5157	0.3094
R ₄	0.4196	0.5157	0.3094
R ₅	0.5874	0.5157	0.7720
R ₆	0.4196	0.3094	0.3094
Weighted Normalized Matrix			
R ₁	0.1242	0.0990	0.0606
R ₂	0.1242	0.0330	0.0606
R ₃	0.2070	0.1649	0.0606
R ₄	0.2070	0.1649	0.0606
R ₅	0.2898	0.1649	0.1414
R ₆	0.2070	0.0990	0.0606

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Table 7 TOPSIS results

Alternatives	D_j^*	D_j^-	CC_j
R ₁	0.1783	0.1043	0.3691
R ₂	0.2118	0.0808	0.2761
R ₃	0.0828	0.1755	0.6794
R ₄	0.0828	0.1755	0.6794
R ₅	0.0808	0.2118	0.7239
R ₆	0.1059	0.1332	0.5571

From table 7 the weightages of the resources are obtained from TOPSIS methodology. Taking into consideration of the weightages of the resources weighted sum of the activities are calculated by using the below formula and ranking the activities according to the weighted sum of the activities.

Where i_j is the value of the resource required for the activity j ; max_j and min_j are the maximum and minimum values of criterion j among all activities.

Table 8 weighted sum of the activities and ranking of the activities

Activity	Duration (days)	Predecessors	Resource Requirements per day						Weighted sum W_s	Rank
			R ₁	R ₂	R ₃	R ₄	R ₅	R ₆		
			W_1	W_2	W_3	W_4	W_5	W_6		
			0.3691	0.2761	0.6794	0.6794	0.7239	0.5571		
A	6	-	5	2	2	2	7	4	1.0056	19
B	3	-	3	5	2	3	9	6	1.4735	10
C	4	A	2	4	4	2	3	1	0.9215	20
D	6	-	5	4	3	5	5	4	1.6707	3
E	7	A,B	3	5	2	3	8	0	1.2523	15
F	5	C	4	1	4	9	2	5	1.6441	4
G	2	D	4	1	4	3	9	8	2.124	1
H	2	A,B	5	5	4	0	9	1	1.7321	2
I	2	G,H	3	2	4	3	4	2	1.152	17
J	6	F	1	5	4	6	7	3	1.374	12
K	1	C,E	3	3	2	4	5	1	1.603	7
L	2	E,G,H	3	2	2	8	3	4	1.2256	16
M	4	I,K	2	2	2	2	4	8	1.0623	18
N	2	F,L	1	4	4	3	4	1	1.6346	5
O	3	L	5	5	4	6	2	3	1.5914	8
P	5	J,M,N	3	2	3	4	7	8	1.6278	6
Q	8	O	4	5	4	2	3	4	1.3707	13
R	2	D,O	5	3	3	3	7	8	1.4923	9
S	6	P,R	2	4	6	2	3	4	1.4702	11



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T	2	Q	1	6	2	7	5	2	1.254	14
Daily Resource Limit			7	10	10	16	18	13		

Table 9 Arranging the activities as per their rank in the ascending order and scheduling the activities as per the precedence relationship by considering the weighted sum ranking.

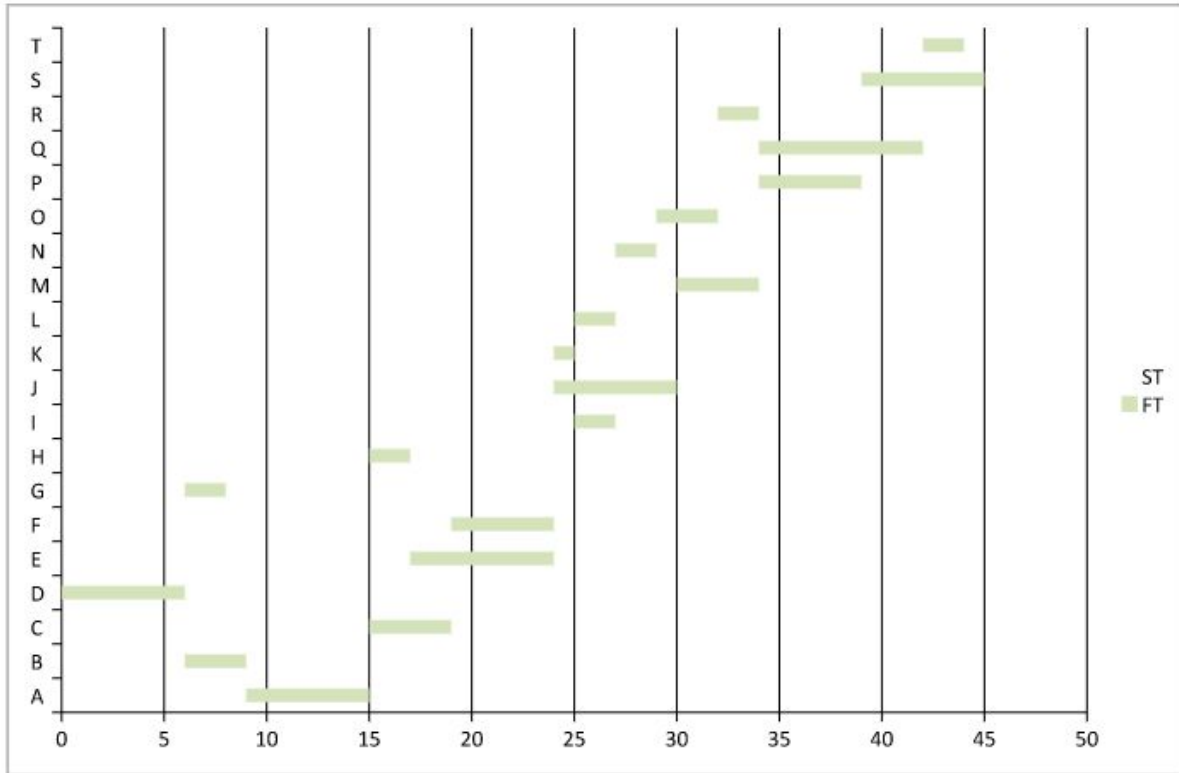
Rank	Activity	Predecessor	Duration (days)	R ₁	R ₂	R ₃	R ₄	R ₅	R ₆
1	G	D	2	4	1	4	3	9	8
2	D	-	6	5	4	3	5	5	4
3	P	J,M,N	5	3	2	3	4	7	8
4	B	-	3	3	5	2	3	9	6
5	J	F	6	1	5	4	6	7	3
6	H	A,B	2	5	5	4	0	9	1
7	R	D,O	2	5	3	3	3	7	8
8	F	C	5	4	1	4	9	2	5
9	O	L	3	5	5	4	6	2	3
10	S	P,R	6	2	4	6	2	3	4
11	A	-	6	5	2	2	2	7	4
12	T	Q	2	1	6	2	7	5	2
13	E	A,B	7	3	5	2	3	8	0
14	Q	O	8	4	5	4	2	3	4
15	K	C,E	1	3	3	2	4	5	1
16	L	E,G,H	2	3	2	2	8	3	4
17	I	G,H	2	3	2	4	3	4	2
18	M	I,K	4	2	2	2	2	4	8
19	N	F,L	2	1	4	4	3	4	1
20	C	A	4	2	4	4	2	3	1



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Daily resource limit	7	10	10	16	18	13
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Gantt chart representing the project schedule



Critical path: D-B-A-C-F-J-M-P-S

Project duration: 45 days

CONCLUSION

The perceptive ability and methods to make sound decisions are involved in the complex decision making situation in the project management. The project scheduling with the help of AHP with TOPSIS in developing the weightages for the activities are done in this paper and the corresponding project schedule is shown in this paper. This paper provides the basis for applications of AHP with TOPSIS weightages in the project scheduling. The final project scheduling and the project duration are obtained from the Gantt chart.

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