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NONLINEAR FINANCIAL MODELING USING DISCRETE FRACTIONAL-ORDER DIFFERENTIATION: AN EMPIRICAL ANALYSIS

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Keywords: *Nonlinear, Dynamic, Discrete financial model, Fractional-order, Least square principle*

ABSTRACT

In this study, a nonlinear financial model of economic dynamics with fractional-order derivative is proposed. Jumari type fractional-order derivative is used to make it discretization by removing the limit operation. We estimate the coefficients and parameters of the model by using least square principle. This new approach to financial system modelling is illustrated by an application to model the behaviour of Indian national financial system which consists of three economic dynamics like interest rate, investment and inflation. The empirical results with different time step sizes discretization are shown; a comparison of the actual data against the data estimated by empirical model is illustrated. The comparative analysis of integer order derivative financial and the present model is shown which gives interesting facts about model. Application of fractional-order discrete technique in nonlinear financial model is a powerful tool for minimizing the error and forecasting of economic dynamics as well as for optimization of management strategy and decision technology of Indian financial system.

INTRODUCTION

The original ideas of fractional calculus traced back to the end of the seventeenth century when the classical differential and integral calculus theories were created and developed by Newton and Leibniz [13]. In the early ages of modern differential calculus, right after the introduction of $\frac{d}{dt}$ for the first derivative, in a later dated 1965, l'Hospital asked Leibniz the meaning of $\frac{d^{\frac{1}{2}}}{dt^{\frac{1}{2}}}$, the derivative of order $\frac{1}{2}$. The appearance of $\frac{1}{2}$ as a fraction gave the name fractional calculus to the study of derivatives, and integrals, of any order, real or complex. So, the fractional calculus, an active branch of mathematics analysis, is as old as the classical calculus which we know today. Fractional calculus is a 310-year-old mathematical topic. Fractional models have been shown by many researchers to describe the applications in physical and biological processes and systems. Applications include visco-elasticity, electro-analytical chemistry, electrical- conductance of biological systems, modelling of neurons, diffusion processes, damping laws and rheology. Recently, research has been applied fractional derivatives to specific fields of research: psychological and life sciences [19]. Since the concepts and calculus of fractional differential equations are several centuries old, it has not been realized until some decades ago that these derivatives could be employed in modelling the real world excellently. Although it has a long history, it was not used in physics, engineering or finance for many years. However, during the past 20 years, interest has been growing in fractional calculus not only among mathematicians but also among physicists, engineers and financial analysts. An extension of fractality concepts in financial mathematics has also been developed [14]. Practically no area of classical analysis has been left untouched by fractional calculus.

Thus, in recent thirty five years, more and more scientists and analysts are attracted to study fractional differential equations in physics, chemistry, engineering, life sciences, finance, and other sciences. Most results about solving fractional differential equations are obtained by using numerical methods, because only some particular fractional differential equations can be solved analytically. The complex dynamics of the financial and economical system are evidenced by large-amplitude and periodic fluctuations, which have attracted much attention in recent times [12, 15, 16, 17]. So, studying the dynamics of financial behaviour it is very difficult to identify the multicollinearity between different financial variables and economic variables quantitatively. As it is realized that finance and economics always illustrate nonlinearity in recent decade, several nonlinear financial models have been developed to study the periodic, chaotic and memory based behaviours in financial and economic systems. In particular, the complex dynamics of economic cycles using the van der pol model is studied in [2-6,9]. The advantage of employing forced van der pol equation to depict economic variable is that the introduction of a forcing function enables us to model the complex interdependence between an individual national economy and the international economy in an era of globalization and the impact of climate variations

ISSN 2349-6193
Impact Factor: 2.805

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like the annual solar cycle on seasonal fluctuations of various financial markets. These studies together with many recent researches lead the studying of the nonlinear dynamics of financial and economic systems to make it more attractive and more accuracy. All the previous researches are related to study the periodic and chaotic behaviours of nonlinear dynamics of financial and economic systems. [10-11, 20] developed a simplified macro-financial model of interest rate, investment demand and inflation and analyzed the balance, stable periodic, chaotic motion, and so forth. In [8], the complex motion in nonlinear dynamic systems by considering the Goodwin's nonlinear accelerator model with periodic investment outlays is studied. It is realized that temporary chaotic dynamics are widespread in nonlinear economic model. In broad sense, as the chaotic dynamics has adverse effect on predicting economy reasonably and effectively, more and more related researches focus on analysing and controlling the chaotic behaviour of nonlinear dynamics in financial and economic systems.

This study determines the suitable parameters and optimal fractional orders by considering in time step sizes of dicretization of financial systems. One of the major differences between fractional order and integer order models is that fractional-order models depends on the history of the system (memory). The magnitude of the financial variables such as foreign exchange rates, gross domestic product, interest rates, production, unemployment and stock market prices can have very long memory i.e. history of the system. It indicates the correlations overlap with the longest time scales in the financial market [18]. It means that all the fluctuations in financial variables are correlated with all future fluctuations. We construct a new discrete financial model, we can analyze the effectiveness of fitting of the empirical financial model. To the authors' knowledge, this is the first time that the realistic financial and economic data of country particularly India are modelled by using continuous or discrete factional-order dynamic financial model. It is hoped that this paper will trigger more research efforts in this direction.

This paper is organised as follows: in Section 2, the mathematical preliminaries are introduced. In Section 3, we present the integer-order financial model recently reported in the literature, and introduce its fractional version i.e. the nonlinear dynamic econometric models of financial system and we estimate the parameters by using the principle of least square. In Section 4, the empirical analysis on Indian economic and financial data from 1981 to 2015 has been done in order to depict the dynamic behaviour by using discretized fractional-order optimization and estimation of nonlinear financial model. Finally, in Section 5 concluding comments are given. Table 1 presents a summary of the results of related researches in recent.

Table 1: Summary of Some Empirical Research on Financial Modelling

Author(s)	Period & Market		Characteristics	Principle	Comparative Analysis	Model
Chian et al. (2005)	-	Chian et al. Type	Order, Chaos, unstable periodic orbits, Chaotic saddles & Intermittency	Forced oscillator, saddle node bifurcation	No	Nonlinear Dynamic
Chen (2006)	-	Caputo-Type	Chaos	Chaos	No	Nonlinear Fractional-order
Xu et al. (2011)	-	Abel Differential Equation	Memory	Short Memory	No	Nonlinear Fractional-order
Yue et al. (2013)	1980-2011, Japan	Jumari Type	Dynamic behaviour	Least Square	No	Nonlinear Fractional-order
Present Study (2017)	1981-2014, India	Jumari Type	Dynamic behaviour	Least Square	Yes	Nonlinear Fractional-order

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MATHEMATICAL PRELIMINARIES

In this section, we introduce some preliminaries of fractional derivative.

Let $f(x): R \to R$ denote a continuous function, and let $h > 0$ denote a constant dicretization span. The fractional difference of order α $(\alpha \in R, 0 < \alpha \leq 1)$ of $f(x)$ is defined as follows [7].

$$\Delta^{\alpha} f(x) = \sum_{k=0}^{+\infty} (-1)^k \binom{\alpha}{k} f(x + (\alpha - k)h) \quad (1)$$

and then its fractional derivative of order α is defined by $D_t^{\alpha} f(t) = \lim_{h \to 0} \frac{\Delta^{\alpha} f(t)}{h^{\alpha}}$ (2)

In [7], the Jumarie's modified fractional derivative of order α for a continuously differentiable function $u: [0, \infty) \to R$ is defined as:

$$D_t^{\alpha} f(t) = \Gamma(1 + \alpha - m) \lim_{h \to 0} \frac{\Delta^{\alpha} f^m(t)}{h^{\alpha - m}}, \quad (3)$$

where $m < \alpha < m + 1, m = [\alpha], [\alpha]$ denotes the integer part of the real number α. Furthermore, if $0 < \alpha < 1$, then

$$D_t^{\alpha} f(t) = \Gamma(1 + \alpha) \lim_{h \to 0} \frac{\Delta^{\alpha} f(t)}{h^{\alpha}} \quad (4)$$

There are several definitions of fractional derivative in [1], but in this paper we only use the Jumarie's fractional derivative. The advantage of applying Jumaie's fractional derivative is that we can select a small step size h and remove the limit operation in (4); then, the discrete form of fractional derivative can be represented by the classical difference of function, multiplied by some coefficients. Now the continuous and discrete financial models are constructed by using Jumaie's fractional derivative.

MODEL DESCRIPTION AND ESTIMATION METHODOLOGY

[11] have recently reported a dynamic model of finance, composed of three first order differential equations. The model describes the time-variation of three state variables: the interest rate, X, the investment demand, Y, and the price index, Z. The factors that influence changes in X mainly come from two aspects: first, contradictions from the investment market, i.e., the surplus between investment and savings, and second, structural adjustment from good prices. The changing rate of Y is in proportion to the rate of investment, and in proportion to an inversion with the cost of investment and interest rates. Changes in Z, on the one hand are controlled by a contradiction between supply and demand in commercial markets, and on the other hand, are influenced by inflation rates. By choosing an appropriate coordinate system and setting appropriate dimensions for every state variable, [11] offer the simplified finance model as:

$$\begin{aligned} \dot{X} &= Z + (Y - a)X, \\ \dot{Y} &= 1 - bY - X^2, \\ \dot{Z} &= -X - cZ, \end{aligned} \quad (5)$$

where a is the saving amount, b is the cost per investment, and c is the elasticity of demand of commercial markets. It is obvious that all the three constants a, b and c are nonnegative coefficients with economic interpretations.

Here, we consider the generalization of system (5) for the fractional incommensurate-order model which takes the form:

$$\begin{aligned} D_t^{q_1} X &= \frac{d^{q_1} X}{dt^{q_1}} = Z + (Y - a)X, \\ D_t^{q_2} Y &= \frac{d^{q_2} X}{dt^{q_2}} = 1 - bY - X^2, \\ D_t^{q_3} Z &= \frac{d^{q_3} X}{dt^{q_3}} = -X - cZ \end{aligned} \quad (6)$$

where, $q_i \in (0,1]$ $(i = 1,2,3)$ represent the fractional order of the derivatives. If $q_1 = q_2 = q_3 = 1$, (6) reduces to the integer-order Chen system. Again the system (6) can written in the form of time variable t.

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$$D_t^{q1}x_t = \frac{d^{q1}x_t}{dt^{q1}} = z_t + (y_t - a)x_t,$$
$$D_t^{q2}y_t = \frac{d^{q2}y_t}{dt^{q2}} = 1 - by_t - {x_t}^2, \quad (7)$$
$$D_t^{q3}z_t = \frac{d^{q3}z_t}{dt^{q3}} = -x_t - cz_t$$

where, $x, y \; and \; z$ represent the interest rate, investment, and inflation respectively. The subscript t indicates that the variable depends on t. Instead of considering the same expressions in fractional chaotic Chen system, we assume a more general form of the present financial model as:

$$D_t^{q1}x_t = c_1 + a_{11}x_t + a_{12}y_t + a_{13}z_t + a_{14}x_t^2 + a_{15}y_t^2 + a_{16}z_t^2 + a_{17}x_ty_t + a_{18}y_tz_t + a_{19}z_tx_t + u_{1t} = \text{The rate of change in investment with fractional order w.r.to time } t,$$

$$D_t^{q2}y_t = c_2 + a_{21}x_t + a_{22}y_t + a_{23}z_t + a_{24}x_t^2 + a_{25}y_t^2 + a_{26}z_t^2 + a_{27}x_ty_t + a_{28}y_tz_t + a_{29}z_tx_t + u_{2t} = \text{The rate of change in interest rate with fractional order w.r.to time } t, \quad (8)$$

$$D_t^{q3}z_t = c_3 + a_{31}x_t + a_{32}y_t + a_{33}z_t + a_{34}x_t^2 + a_{35}y_t^2 + a_{36}z_t^2 + a_{37}x_ty_t + a_{38}y_tz_t + a_{39}z_tx_t + u_{3t} = \text{The rate of change in inflation with fractional order w.r.to time } t$$

Where $u_{it}(i = 1,2,3)$ are the random errors which are assumed to the white noise generally, $x_t = x(t), y_t = y(t) \; and \; z_t = z(t)$ indicate that the variables $x, y \; and \; z$ depending on time t.

$$f_1(x_t, y_t, z_t, A_1) = c_1 + a_{11}x_t + a_{12}y_t + a_{13}z_t + a_{14}x_t^2 + a_{15}y_t^2 + a_{16}z_t^2 + a_{17}x_ty_t + a_{18}y_tz_t + a_{19}z_tx_t,$$
$$f_2(x_t, y_t, z_t, A_2) = c_2 + a_{21}x_t + a_{22}y_t + a_{23}z_t + a_{24}x_t^2 + a_{25}y_t^2 + a_{26}z_t^2 + a_{27}x_ty_t + a_{28}y_tz_t + a_{29}z_tx_t,$$
(9)

$$f_3(x_t, y_t, z_t, A_3) = c_3 + a_{31}x_t + a_{32}y_t + a_{33}z_t + a_{34}x_t^2 + a_{35}y_t^2 + a_{36}z_t^2 + a_{37}x_ty_t + a_{38}y_tz_t + a_{39}z_tx_t$$

Where $A_i = (c_i, a_{i1}, a_{i2}, \dots \dots \dots \dots .., a_{i9}), i = 1,2,3;$

Then the model can be rewritten as:

$$D_t^{q1}x_t = f_1(x_t, y_t, z_t, A_1) + u_{1t},$$
$$D_t^{q2}y_t = f_2(x_t, y_t, z_t, A_2) + u_{2t}, \quad (10)$$
$$D_t^{q3}z_t = f_3(x_t, y_t, z_t, A_3) + u_{3t}$$

According to (4), when $0 < \alpha < 1$, the model (10) can be discretized as:

$$D_t^{q1}x_t = \frac{x(t_{n+1}) - x(t_n)}{(t_{n+1} - t_n)^{q1}}\Gamma(1 + q_1) = f_1(x_t, y_t, z_t, A_1) + u_{1t},$$
$$D_t^{q2}y_t = \frac{y(t_{n+1}) - y(t_n)}{(t_{n+1} - t_n)^{q2}}\Gamma(1 + q_2) = f_2(x_t, y_t, z_t, A_2) + u_{2t} \quad (11)$$
$$D_t^{q3}z_t = \frac{z(t_{n+1}) - z(t_n)}{(t_{n+1} - t_n)^{q3}}\Gamma(1 + q_3) = f_3(x_t, y_t, z_t, A_3) + u_{3t}$$

We estimate (11) based on empirical data to determine the relationship of these variables. From the form of the model (11), it is easy to find that there does not exists common parameters in three equations of it. Therefore, the above three multivariate regression equations can be estimated separately. To state the technical procedures, we take the first equation as an example. The estimation for the parameters in the other two equations is similar.

Consider $q_1 = 1$; then

$$\frac{x(t_{n+1}) - x(t_n)}{(t_{n+1} - t_n)} = f_1(x_t, y_t, z_t, A_0) + u_{1t} \quad (12)$$

Let $Y_i = \frac{x(t_{i+1}) - x(t_i)}{(t_{i+1} - t_i)}$, i = 1,2,……. N − 1; then define the least squares (LS) function as:

$$SSR\,(A_0) = \sum_{i=1}^{N-1}(Y_i - f_1(x(t_i), y(t_i), z(t_i), A_0)^2 \quad (13)$$

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The LS estimator of the regression parameter $\widehat{A_0} = \arg\min \sum_{i=1}^{N-1}(Y_i - f_1(x(t_i), y(t_i), z(t_i), A_0)^2$ (14)

For simplicity, we denote $X_{1i} = x(t_i), X_{2i} = y(t_i), X_{3i} = z(t_i), X_{4i} = x(t_i)^2, X_{5i} = y(t_i)^2, X_{6i} = z(t_i)^2, X_{7i} = x(t_i)y(t_i), X_{8i} = y(t_i)z(t_i), X_{9i} = z(t_i)x(t_i), i = 1,2, \ldots\ldots, N-1$, where N is the number of sample studied. Similar to the procedure of estimating the multivariate regression by the method of least squares, we can obtain the least squares estimator of the model as:

$$\widehat{A_0} = (X^T X)^{-1}(X^T Y), \quad (15)$$

where

$$X = \begin{pmatrix} 1 & X_{11} & X_{12} & \cdots & X_{19} \\ 1 & X_{21} & X_{22} & \cdots & X_{29} \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ 1 & X_{(N-1)1} & X_{(N-1)2} & \cdots & X_{(N-1)N} \end{pmatrix} \quad (16)$$

and $Y = (Y_1, Y_2, \ldots\ldots, Y_{N-1})^T$. The subscript T indicates the transportation of matrix and vector.

We consider the first regression equation and estimate the parameters (q_1, A_1). The corresponding least squares estimation is subjected to:

$$(\widehat{q_1}, \widehat{A_1}) = \arg\min SSR\ (q_1, A_1) = \arg\min \sum_{n=1}^{N-1}\left(\frac{x(t_{n+1}) - x(t_n)}{(t_{n+1} - t_n)^{q_1}} \Gamma(1+q_1) - f_1(x(t_n), y(t_n), z(t_n), A_1)\right)^2 = \arg\min\ (\Gamma(1+q_1)(t_{n+1} - t_n)^{1-q_1})^2 \times \sum_{n=1}^{N-1}\left(\frac{x(t_{n+1}) - x(t_n)}{(t_{n+1} - t_n)} - f_1(x(t_n), y(t_n), z(t_n), A_1^{'})\right)^2, \quad (17)$$

where $A_1^{'} = \frac{(t_{n+1} - t_n)^{q_1 - 1} A_1}{\Gamma(1+q_1)}$ (18)

According to the evaluation result with $q_1 = 1$, the minimum of the second product part implies that $A_1^{'} = \widehat{A_0}$. Hence,

$$\widehat{q_1} = \arg\min\ \{\Gamma(q_1 + 1)(t_{n+1} - t_n)^{1-q_1}\}, A_1^{'} = \widehat{A_0}, \quad (19)$$

and the minimum of $SSR\ (q_1, A_1)$ can be obtained as:

$$\widehat{A_1} = \Gamma(\widehat{q_1} + 1)(t_{n+1} - t_n)^{1-\widehat{q_1}}(X^T X)^{-1}(X^T Y), \quad (20)$$

$$\widehat{q_1} = \arg\min\ \{\Gamma(q_1 + 1)(t_{n+1} - t_n)^{1-q_1}\} \quad (21)$$

Equations (20)-(21) are the least squares estimation of q_1 and A_1 in the first regression equation of system (11), respectively. It is easy to find that the estimator of $q_1$is not related to the sample observations, and can be computed by numerically. Using the same technique, we can deal with q_2 and q_3 in system (11) and obtain the optimal estimators of $\widehat{q_2}$ and $\widehat{q_3}$. In the next section, the dynamics of the new model and prediction results based on the macroeconomic data of India is considered.

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EMPIRICAL RESULTS OF DISCRETE FINANCIAL SYSTEM: EVIDENCE FROM INDIA

In this section, we present the study of discrete financial system based on the macroeconomic data of India.

Data Description

In financial model (11), the nonlinear dynamic behaviours of interest rate, investment demand and inflation are studied. This work chooses six-month London interbank offered rate (LIBOR) data to reflect interest rate change in India. The total investment percent of GDP is used to measure the investment demand. Average consumer prices percent change rate will be used to reflect the inflation. The annual data starts from year 1981 to 2015. The data about LIBOR, investment percent of GDP, and average consumer prices percent rate are obtained from EconStats which is organized by IMF.

Empirical Results

The optimal fractional orders $q_i = 1,2,3$ with different step sizes $\Delta t = 1, 0.9, 0.8, 0.7, 0.6$ are performed in Table 2. We do not consider the case of $\Delta t < 0.5$ because of the fact that the fractional order decreases and approaches to zero, which reduces this present model to be a linear one but not the fractional financial system.

$\widehat{Y}_i = 7.456 - 1.379x_t + 0.343y_t - 2.781z_t + 0.02x_t^2 + 0.005y_t^2 + 0.155z_t^2 + 0.066x_ty_t - 0.025y_tz_t + 0.064z_tx_t,$

$\widehat{Y}_i = 7.456 - 1.379x_t + 0.343y_t - 2.781z_t + 0.02x_t^2 + 0.005y_t^2 + 0.155z_t^2 + 0.066x_ty_t - 0.025y_tz_t + 0.064z_tx_t,$

$\widehat{Y}_i = 7.456 - 1.379x_t + 0.343y_t - 2.781z_t + 0.02x_t^2 + 0.005y_t^2 + 0.155z_t^2 + 0.066x_ty_t - 0.025y_tz_t + 0.064z_tx_t,$

Table 2: Optimal fractional order q

Δt	1	0.9	0.8	0.7	0.6	0.5
Extended Solver Steps / Solver Iteration	2/18	2/17	2/18	2/14	2/16	2/12
$\widehat{q}_i$	0.4616321	0.3579855	0.2536069	0.1482567	0.04157812	0
W	0.8856032	0.8321360	0.7667172	0.6890221	0.5991806	0.5

Note: When$\Delta t \leq 0.5$, $\widehat{q_i} \leq 0$; thus the case of $\Delta t \leq 0.5$ is not to be considered and $W = \Gamma(\widehat{q}_1 + 1)\Delta t1 - q1$.

Table 3: The first equation in () under the several time steps of dicretization

Parameters: Coefficients of the variables	Δt: 1	0.9	0.8	0.7	0.6	1
$\widehat{q}_i$	0.4616321	0.3579855	0.2536069	0.1482567	0.04157812	1
c	6.603 (0.617)	6.204 (0.617)	5.716 (0.617)	5.137 (0.617)	4.467 (0.617)	7.456 (0.617)
x_t	-1.221 (0.183)	-1.147 (0.183)	-1.057 (0.183)	-0.950 (0.183)	-0.826 (0.183)	-1.379 (0.183)
y_t	0.303 (0.605)	0.285 (0.605)	0.263 (0.605)	0.236 (0.605)	0.205 (0.605)	0.343 (0.605)
z_t	-2.463 (0.472)	-2.314 (0.472)	- 2.132(0.472)	-1.916 (0.472)	-1.666 (0.472)	-2.781 (0.472)
x_t^2	0.018 (0.043)	0.017 (0.043)	0.016 (0.043)	0.014 (0.043)	0.012 (0.043)	0.02 (0.043)
y_t^2	0.005 (0.867)	0.004 (0.867)	0.004 (0.867)	0.004 (0.867)	0.003 (0.867)	0.005 (0.867)
z_t^2	0.137 (0.511)	0.129 (0.511)	0.119 (0.511)	0.107 (0.511)	0.093 (0.511)	0.155 (0.511)

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$x_t y_t$	0.058 (0.156)	0.055 (0.156)	0.051 (0.156)	0.045 (0.156)	0.039 (0.156)	0.066 (0.156)
$y_t z_t$	-0.022 (0.699)	-0.021 (0.699)	-0.019 (0.699)	-0.017 (0.699)	-0.015 (0.699)	-0.025 (0.699)
$z_t x_t$	0.056 (0.584)	0.053 (0.584)	0.049 (0.584)	0.044 (0.584)	0.038 (0.584)	0.064 (0.584)
R^2	0.681	0.681	0.681	0.681	0.681	0.681
SSR	1393.396	1230.226	1044.399	843.456	637.840	1776.627
$Prob(F)$	0.000	0.000	0.000	0.000	0.000	0.000

Note. Bracketed value denotes the statistical significance at the 5% levels. R^2 is the coefficient of determination, SSR is the sum of squared residuals, $Prob(F)$ is the P value of F- statistic.

Table 4: The first equation in () under the several time steps of dicretization

Parameters: Coefficients of the Variables	Δt					
	1	0.9	0.8	0.7	0.6	1
	$\hat{q}_i$					
	0.4616321	0.3579855	0.2536069	0.1482567	0.04157812	1
c	5.212 (0.632)	4.897 (0.632)	4.512 (0.632)	4.055 (0.632)	3.526 (0.632)	5.885 (0.632)
x_t	0.432 (0.561)	0.406 (0.561)	0.374 (0.561)	0.336 (0.561)	0.292 (0.561)	0.488 (0.561)
y_t	-0.689 (0.162)	-0.647 (0.162)	-0.596 (0.162)	-0.536 (0.162)	-0.466 (0.162)	-0.778 (0.162)
z_t	-0.778 (0.782)	-0.731 (0.782)	-0.674 (0.782)	-0.605 (0.782)	-0.526 (0.782)	-0.879 (0.782)
x_t^2	-0.005 (0.446)	-0.005 0.446)	-0.005 0.446)	-0.004 0.446)	-0.004 0.446)	-0.006 (0.446)
y_t^2	0.018 (0.437)	0.017 (0.437)	0.015 (0.437)	0.014 (0.437)	0.012 (0.437)	0.02 (0.437)
z_t^2	0.025 (0.886)	0.023 (0.886)	0.021 (0.886)	0.019 (0.886)	0.017 (0.886)	0.028 (0.886)
$x_t y_t$	-0.019 (0.562)	-0.018 (0.562)	-0.017 (0.562)	-0.015 (0.562)	-0.013 (0.562)	-0.022 (0.562)
$y_t z_t$	-0.046 (0.33)	-0.043 (0.33)	-0.040 (0.33)	-0.036 (0.33)	-0.031 (0.33)	-0.052 (0.33)
$z_t x_t$	-0.063 (0.461)	-0.059 (0.461)	-0.054 (0.461)	-0.049 (0.461)	-0.042 (0.461)	-0.071 (0.461)
R^2	0.618	0.618	0.618	0.618	0.618	0.618
SSR	944.441	833.845	707.892	571.693	432.327	1204.194
$Prob(F)$	0.000	0.000	0.000	0.000	0.000	0.000

Note. Bracketed value denotes the statistical significance at the 5% levels. R^2 is the coefficient of determination, SSR is the sum of squared residuals, $Prob(F)$ is the P value of F- statistic.

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Table 5: The first equation in () under the several time steps of dicretization

Parameters: Coefficients of the Variables	Δt					
	1	0.9	0.8	0.7	0.6	1
	$\hat{q}_i$					
	0.4616321	0.3579855	0.2536069	0.1482567	0.04157812	1
c	-2.93 (0.46)	-2.753 (0.46)	-2.537 (0.46)	-2.28 (0.46)	-1.983 (0.46)	-3.309 (0.46)
x_t	-0.255 (0.349)	-0.239 (0.349)	-0.22 (0.349)	-0.198 (0.349)	-0.172 (0.349)	-0.287 (0.349)
y_t	-0.012 (0.945)	-0.011 (0.945)	-0.01 (0.945)	-0.009 (0.945)	-0.008 (0.945)	-0.014 (0.945)
z_t	1.413 (0.174)	1.328 (0.174)	1.223 (0.174)	1.099 (0.174)	0.956 (0.174)	1.595 (0.174)
x_t^2	0.003 (0.292)	0.003 (0.292)	0.002 (0.292)	0.002 (0.292)	0.002 (0.292)	0.003 (0.292)
y_t^2	-0.003 (0.671)	-0.003 (0.671)	-0.003 (0.671)	-0.003 (0.671)	-0.002 (0.671)	-0.004 (0.671)
z_t^2	-0.12 (0.063)	-0.113 (0.063)	-0.104 (0.063)	-0.093 (0.063)	-0.081 (0.063)	-0.135 (0.063)
$x_t y_t$	0.001 (0.967)	0.000 (0.967)	0.000 (0.967)	0.000 (0.967)	0.000 (0.967)	0.001 (0.967)
$y_t z_t$	0.008 (0.644)	0.007 (0.644)	0.007 (0.644)	0.006 (0.644)	0.005 (0.644)	0.009 (0.644)
$z_t x_t$	0.032 (0.311)	0.03 (0.311)	0.027 (0.311)	0.025 (0.311)	0.021 (0.311)	0.036 (0.311)
R^2	0.546	0.546	0.546	0.546	0.546	0.546
SSR	124.866	110.244	93.592	75.585	57.159	159.209
$Prob(F)$	0.011	0.011	0.011	0.011	0.011	0.011

Note. Bracketed value denotes the statistical significance at the 10% levels. R^2 is the coefficient of determination, SSR is the sum of squared residuals, $Prob(F)$ is the P value of F- statistic.

Table 3-5 show the results about the estimated coefficients, coefficient of determination, sum of residuals, and P values of statistical tests in equations of empirical model under the situation of different time steps of discretization. For comparative purpose the results of the estimated coefficients and other values also given under integer-order Chen system.

In Table 3 it is observed that the R^2 (coefficient of determination) value is 0.681which shows that 68.1% of the variation in interest rate can be explained by the nine independent variables. The significant F is 0.0001. This indicates that the model is statistically significant at a confidence level 99.999%. We also have the $t-test$ value for the significance of individual independent variables to indicate the significance level at 95% confidence level. From this table it is observed that only the dynamic independent variable x_t^2 is statistically significant with a value of 0.043 which is less than 0.05 (significant). The other eight dynamic independent variables are individually not significant. Finally, the structure of terms at which corresponding coefficients estimated are significant at 5% level in the empirical equation about interest rate includes terms $x, y, z, x^2, y^2, z^2, xy, yz, zx, constant\ term$ and is independent of the time step size discretization. The result is consistent with the fact that the estimation of q_i is not related to the sample observations in (21). The sum of square due to residual of the integer-order nonlinear financial model of dynamic interest rate is comparatively more than that of all the fractional-order of different steps. So the fractional nonlinear systems offer greater insights towards understanding the dynamic behaviour of Indian financial system.

In Table 4 it is observed that the R^2 (coefficient of determination) value is 0.618 which shows that 61.8% of the variation in investment can be explained by the nine independent variables. The significant F is 0.0001. This indicates that the model is statistically significant at a confidence level 99.999%. We also have the $t-$

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$test$ value for the significance of individual independent variables to indicate the significance level at 95% confidence level. From this table it is observed that all nine dynamic independent variables are individually not significant since all the significant values are more than 0.05 (insignificant). Finally, the structure of terms at which corresponding coefficients estimated are significant at 5% level in the empirical equation about investment rate includes terms $x, y, z, x^2, y^2, z^2, xy, yz, zx, constant\ term$ and is independent of the time step size discretization. The result is consistent with the fact that the estimation of q_i is not related to the sample observations in (21).The sum of square due to residual of the integer-order nonlinear financial model of dynamic investment is comparatively more than that of all the fractional-order of different steps. So the fractional nonlinear systems offer greater insights towards understanding the dynamic behaviour of Indian financial system.

In Table 5 it is observed that the R^2 (coefficient of determination) value is 0.546 which shows that 54.6% of the variation in inflation can be explained by the nine independent variables. The significant F is 0.0001. This indicates that the model is statistically significant at a confidence level 99.999%. We also have the $t-test$ value for the significance of individual independent variables to indicate the significance level at 90% confidence level. From this table it is observed that only the dynamic independent variable z_t^2 is statistically significant with a value of 0.063 which is less than 0.1 (significant). The other eight dynamic independent variables are individually not significant. Finally, the structure of terms at which corresponding coefficients estimated are significant at 5% level in the empirical equation about inflation includes terms $x, y, z, x^2, y^2, z^2, xy, yz, zx, constant\ term$ and is independent of the time step size discretization. The result is consistent with the fact that the estimation of q_i is not related to the sample observations in (21).The sum of square due to residual of the integer-order nonlinear financial model of dynamic inflation is comparatively more than that of all the fractional-order of different steps. So the fractional nonlinear systems offer greater insights towards understanding the dynamic behaviour of Indian financial system.

Figures 1-3 show the data estimated by the empirical model in case of $\Delta t = 0.6$ and the actual data of the interest rate, investment and inflation respectively. In Figure 1, we can find that black line passes through the vast majority of the circle. According to the circles and black line which depict the estimated data and actual data of the interest rate respectively, we find that the empirical equation of the interest rate in the model can describe the actual data effectively. In Figure 2, the black line almost passes through maximum of the circle notations. The property of the figure represents that the actual investment data can be fitted empirical equation ideally. In Figure 3, there are only very few circle notations that deviate from the black line in the small part of beginning of the period. Figure 3 suggests that the empirical equation about inflation fits the actual data accurately. Above all, Figures 1-3 suggest that the empirical model estimated by the methodology proposed in the paper depicts the actual data practically.

In order to analyze the effectiveness of the consequence about the prediction of the empirical model, the multistep about the prediction of interest rate, investment and inflation in the case that $\Delta t = 1, 0.9, 0.8, 0.7, 0.6\ and\ (\Delta t = 1\ and\ q = 1)$ are shown in Figures 4-6, respectively. From Figure 4, we can find that the predictions are very close to the actual interest rate. It suggests that the prediction of interest rate is meaningful in case of all the six cases of fractional order and integer first order. From Figure 5, we can find that the predictions are very close to the actual investment. It suggests that the prediction of investment is meaningful in case of all the six cases of fractional order and integer first order. From Figure 6, we can find that the predictions are very close to the actual inflation. It suggests that the prediction of inflation is meaningful in case of all the six cases of fractional order and integer first order. The above results suggest that all the cases of predictions are giving the optimization of fractional-order with suitable Δt of the fitting of the meaningful estimation of empirical model of interest rate, investment and inflation.

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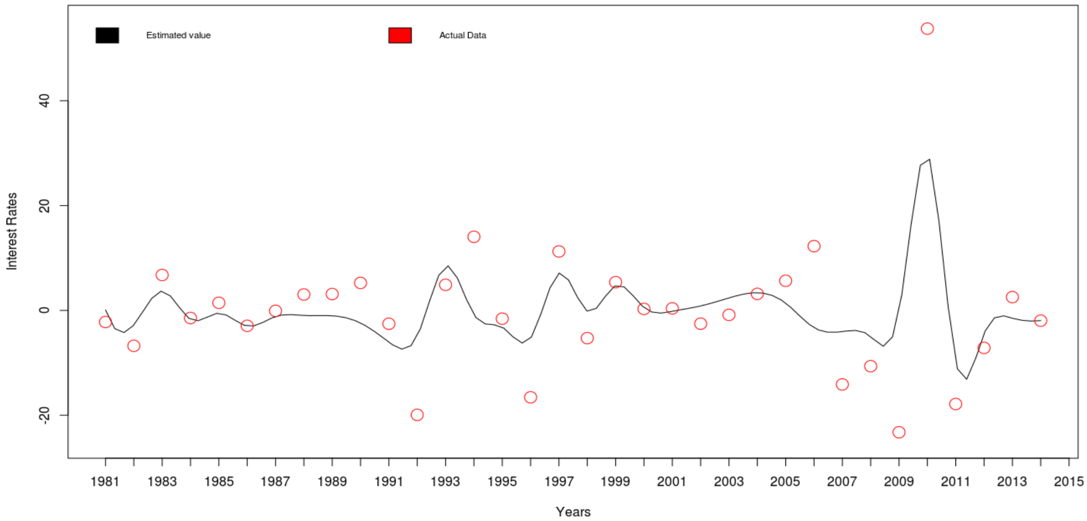

Figure 1: The actual interest rate versus estimated interest rate

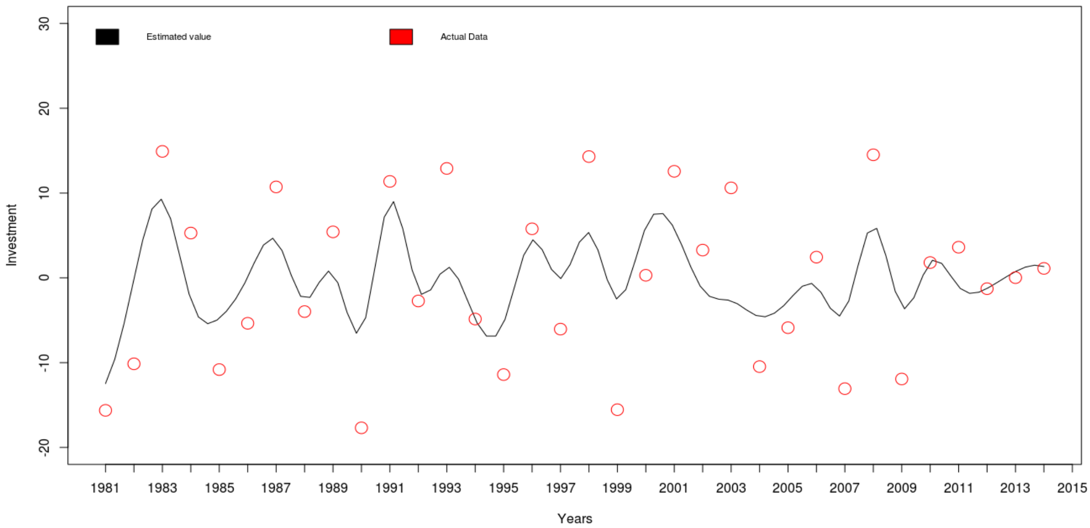

Figure 2: The actual investment versus estimated investment

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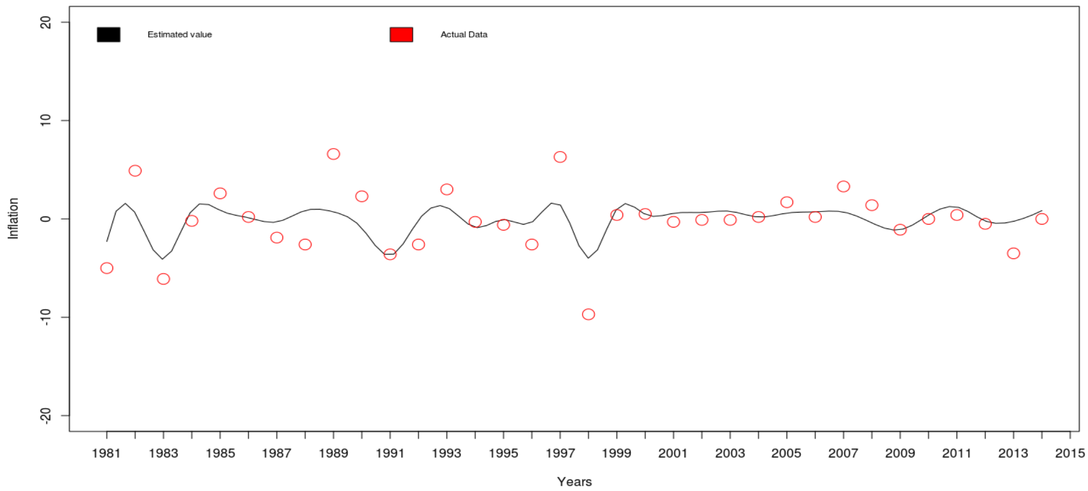

Figure 3: The actual inflation versus estimated inflation

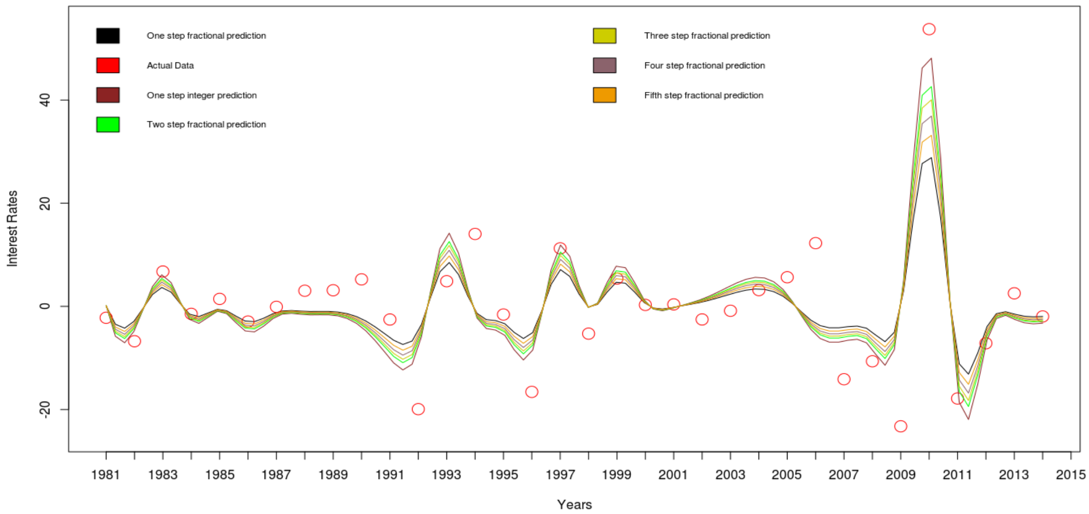

Figure 4: Fractional-order predictions of interest rate on empirical model

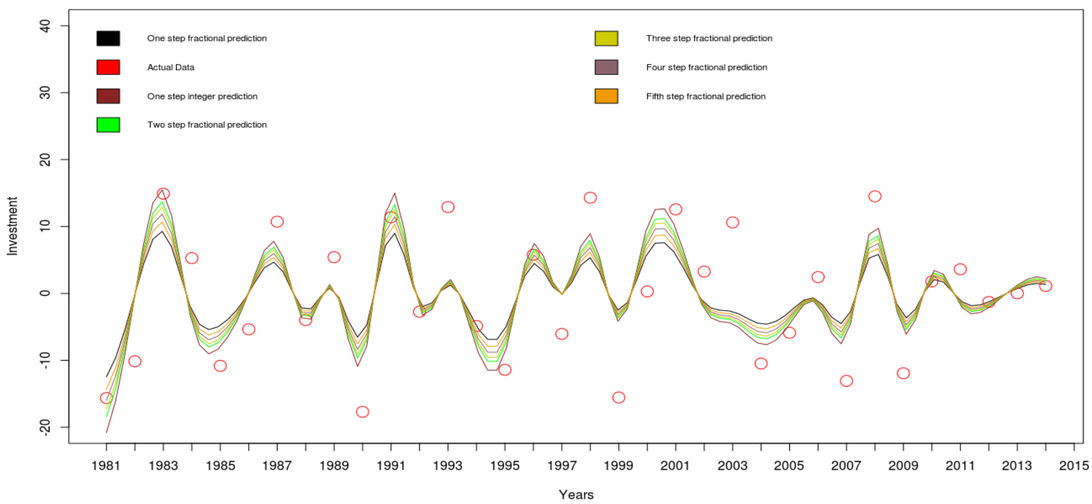

Figure 5: Fractional-order predictions of investment on empirical model

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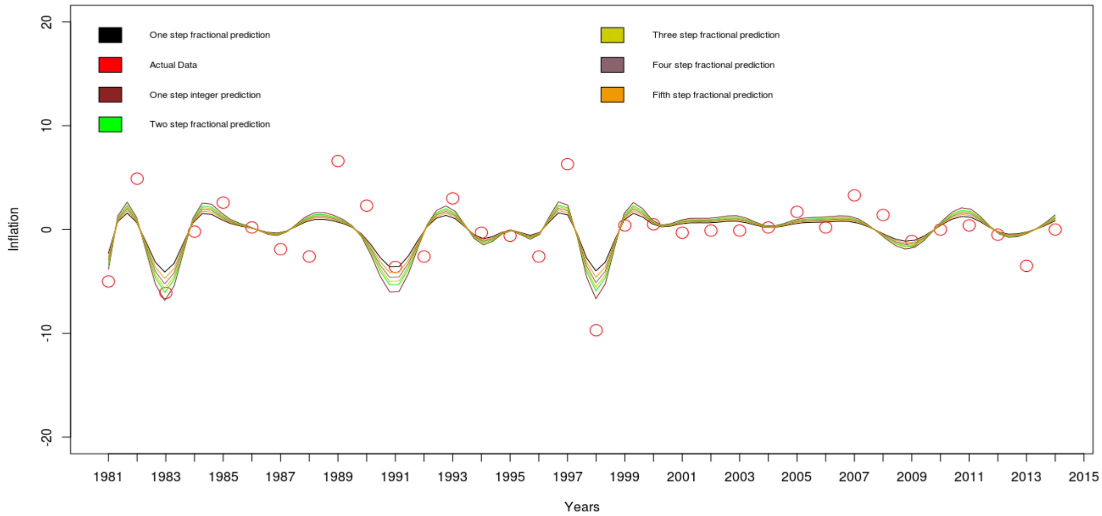

Figure 6: Fractional-order predictions of inflation on empirical model

CONCLUSIONS

A fractional model of finance is proposed as an integer-order model that has recently been reported. In this study we propose a new nonlinear dynamic financial econometric model by using the Jumarie's fractional-order derivative. The corresponding discrete financial model is generated by removing the limit operation in the Jumarie's derivative. The model overcomes the problems which cannot be depicted by some simplified nonlinear financial model in literatures, and it provides a feasible technique for describing the actual macroeconomic data of one particular place by nonlinear model. This present model is proved to be reasonable with empirical analysis.

Based on the macroeconomic data of India, we evaluate the parameters of the dynamic financial model. The suitable fractional order for India's data is obtained. In the empirical study, it is observed that the fractional order has an apparent influence on the dynamics behaviour of financial system. With the optimal fractional order, this new fractional financial model can be used to predict and to offer greater insights towards understanding the dynamic behaviour of financial systems of India in the coming years practically.

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