[Mariapresenti *, 5(7): July, 2018]
DOI: 10.5281/zenodo.1314188
ISSN 2349-6193
Impact Factor: 3.866
IJESMR
International Journal OF Engineering Sciences & Management Research

VAGUE BLOCK MATRIX

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Keywords: *Vague block matrix, vague Kronecker product and vague Kronecker sum.*

ABSTRACT
In this paper the idea t of vague matrix is extended in the field of vague block matrix and discussed some of their relational operations on vague block matrices.

INTRODUCTION
The fuzziness was mathematically described for the first time by L. A. Zadeh[10] in his classical paper in the year 1965. The idea of fuzzy matrix was presented by Thomason[9] plays a vital role in scientific development. The fuzzy matrices has been employed in many approaches to model the diagnostic and decision making process .The fuzzy matrix [3,4,6] have been proposed to represent fuzzy relation in a system based on fuzzy set theory. Many authors have exhibited a number of results on fuzzy matrices. Pal and Shyamal [7,8] introduced two new operators on fuzzy matrices and shown several properties of them. In this paper, we introduce the concept of vague block matrix and defined various forms of vague block matrix and also derived some of the properties of vague direct sum, vague Kronecker sum and vague Kronecker product of vague block matrix.

PRELIMINARIES

Definition 2.1:[2] A vague set A in the universe of discourse U is characterized by two membership functions given by:

(i) A true membership function $t_A : U \to [0,1]$ and

(ii) A false membership function $f_A : U \to [0,1]$

where $t_A(x)$ is a lower bound on the grade of membership of x derived from the "evidence for x", $f_A(x)$ is a lower bound on the negation of x derived from the "evidence for x", and $t_A(x) + f_A(x) \le 1$. Thus the grade of membership of u in the vague set A is bounded by a subinterval $[t_A(x), 1 - f_A(x)]$ of [0,1]. this indicates that if the actual grade of membership of x is μ(x), then, $t_A(x) \le \mu(x) \le 1 - f_A(x)$ .The vague set A is written as $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / u \in U\}$ where the interval $[t_A(x), 1 - f_A(x)]$ is called the vague value of x in A, denoted by $V_A(x)$.

Definition 2.2:[1] Let A and B be VSs of the form $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / x \in X\}$ and $B = \{\langle x, [t_B(x), 1 - f_B(x)]\rangle / x \in X\}$ Then

(i) $A \subseteq B$ if and only if $t_A(x) \le t_B(x)$ and $1 - f_A(x) \le 1 - f_B(x)$ for all $x \in X$

(ii) A=B if and only if $A \subseteq B$ and $B \subseteq A$

(iii) $A^c = \{\langle x, f_A(x), 1 - t_A(x)\rangle / x \in X\}$

(iv) $A \cap B = \{\langle x, \min(t_A(x), t_B(x)), \min(1 - f_A(x), 1 - f_B(x))\rangle / x \in X\}$

(v) $A \cup B = \{\langle x, (t_A(x) \vee t_B(x)), (1 - f_A(x) \vee 1 - f_B(x))\rangle / x \in X\}$

For the sake of simplicity, we shall use the notation $A = \langle x, t_A, 1 - f_A\rangle$ instead of $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / x \in X\}$.

Definition 2.3:[5] A vague matrix A of order $m \times n$ is defined as $A = [x, [< t_{ij}, 1 - f_{ij} >]]_{m \times n}$ e the function $t_{ij} : X \to [0,1]$ and $f_{ij} : X \to [0,1]$ define the degree of truth membership function and the degree of false

 ISSN 2349-6193
Impact Factor: 3.866

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membership function of x_{ij} in A respectively satisfying the condition $0 \le t_{ij} + f_{ij} \le 1$ for all i,j. The value of $\pi_{Aij}(x) = 1 - (t_{Aij}(x) + f_{Aij}(x))$ is called the vague hesitation degree of the element $x \in X$ respectively to the vague matrix A. For simplicity, we write $A = [a_{ij}]_{m\times n}$ where $a_{ij} = [a_{ij(t)}, a_{ij(i-f)}]$.

Vague block matrices:

Definition 3.1: A vague submatrix of a vague matrix of order greater than or equal to 1 is obtained by deleting some rows or some columns or both(not necessarily consecutive) or neither.

Remark 3.2: A vague matrix itself is its vague submatrix. The maximum number of vague submatrices of an $n \times m$ vague matrix is $(2^n - 1)(2^m - 1)$.

Definition 3.3: A vague submatrix of order (n-r) obtained by deleting r rows and columns of an n square vague matrix is called vague principal submatrix. The first order principal vague submatrices obtained from the following third order vague matrix $\begin{bmatrix} [a_{11(t)}, a_{11(1-f)}] & [a_{12(t)}, a_{12(1-f)}] & [a_{13(t)}, a_{13(1-f)}] \\ [a_{21(t)}, a_{21(1-f)}] & [a_{22(t)}, a_{22(1-f)}] & [a_{23(t)}, a_{23(1-f)}] \\ [a_{31(t)}, a_{31(1-f)}] & [a_{32(t)}, a_{32(1-f)}] & [a_{33(t)}, a_{33(1-f)}] \end{bmatrix}$ are $\left[[a_{11(t)}, a_{11(1-f)}]\right]$, $\left[[a_{12(t)}, a_{12(1-f)}]\right]$ and $\left[[a_{13(t)}, a_{13(1-f)}]\right]$. Second order vague submatrices are $\begin{bmatrix} [a_{11(t)}, a_{11(1-f)}] & [a_{12(t)}, a_{12(1-f)}] \\ [a_{21(t)}, a_{21(1-f)}] & [a_{22(t)}, a_{22(1-f)}] \end{bmatrix}$, $\begin{bmatrix} [a_{22(t)}, a_{22(1-f)}] & [a_{23(t)}, a_{23(1-f)}] \\ [a_{32(t)}, a_{32(1-f)}] & [a_{33(t)}, a_{33(1-f)}] \end{bmatrix}$, $\begin{bmatrix} [a_{11(t)}, a_{11(1-f)}] & [a_{13(t)}, a_{13(1-f)}] \\ [a_{31(t)}, a_{31(1-f)}] & [a_{33(t)}, a_{33(1-f)}] \end{bmatrix}$. Third order vague submatrix is the matrix itself.

Definition 3.4: A vague submatrix of order (n-r) obtained by deleting last r rows and columns of an n square vague matrix A is called leading principal vague submatrix. The first order leading principal vague submatrix is $\left[[a_{11(t)}, a_{11(1-f)}]\right]$. The second order leading principal vague submatrix of the above vague matrix is $\begin{bmatrix} [a_{11(t)}, a_{11(1-f)}] & [a_{12(t)}, a_{12(1-f)}] \\ [a_{21(t)}, a_{21(1-f)}] & [a_{22(t)}, a_{22(1-f)}] \end{bmatrix}$.

Definition 3.5: The vague matrix whose elements are blocks obtained by partitioning is called vague block matrix. Here a vague matrix is divided or partitioned into smaller vague matrices called blocks or cells with consecutive rows and column separated by dotted horizontal lines between rows and vertical lines between columns. Thus

$$A = \begin{bmatrix} [a_{11(t)}, a_{11(1-f)}] & [a_{12(t)}, a_{12(1-f)}] & \vdots & [a_{13(t)}, a_{13(1-f)}] & [a_{14(t)}, a_{14(1-f)}] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [a_{21(t)}, a_{21(1-f)}] & [a_{22(t)}, a_{22(1-f)}] & \vdots & [a_{23(t)}, a_{23(1-f)}] & [a_{24(t)}, a_{24(1-f)}] \\ [a_{31(t)}, a_{31(1-f)}] & [a_{32(t)}, a_{32(1-f)}] & \vdots & [a_{33(t)}, a_{33(1-f)}] & [a_{34(t)}, a_{34(1-f)}] \end{bmatrix} = \begin{bmatrix} P_{11} & \vdots & P_{12} \\ \cdots & \vdots & \cdots \\ P_{21} & \vdots & P_{22} \end{bmatrix}$$

where $P_{11} = \left[[a_{11(t)}, a_{11(1-f)}] \quad [a_{12(t)}, a_{12(1-f)}]\right]$, $P_{12} = \left[[a_{13(t)}, a_{13(1-f)}] \quad [a_{14(t)}, a_{14(1-f)}]\right]$, $P_{21} = \begin{bmatrix} [a_{21(t)}, a_{21(1-f)}] & [a_{22(t)}, a_{22(1-f)}] \\ [a_{31(t)}, a_{31(1-f)}] & [a_{32(t)}, a_{32(1-f)}] \end{bmatrix}$ and $P_{31} = \begin{bmatrix} [a_{23(t)}, a_{23(1-f)}] & [a_{24(t)}, a_{24(1-f)}] \\ [a_{33(t)}, a_{33(1-f)}] & [a_{34(t)}, a_{34(1-f)}] \end{bmatrix}$.

ISSN 2349-6193
Impact Factor: 3.866
IJESMR
International Journal OF Engineering Sciences & Management Research

Definition 3.6: The transpose of vague block matrix is the transpose of both blocks and constituent blocks. $A^T = \begin{bmatrix} P_{11}^T & P_{12}^T \\ P_{21}^T & P_{22}^T \end{bmatrix}$.

Definition 3.7: If the number of rows and the number of columns of blocks are equal then the matrix is called square vague block matrix. Thus the partitioned vague matrix,

$$A = \begin{bmatrix} [a_{11(t)}, a_{11(1-f)}] & [a_{11(t)}, a_{11(1-f)}] & \vdots & [a_{11(t)}, a_{11(1-f)}] & [a_{11(t)}, a_{11(1-f)}] & \vdots & [a_{11(t)}, a_{11(1-f)}] & [a_{11(t)}, a_{11(1-f)}] \\ [a_{11(t)}, a_{11(1-f)}] & [a_{11(t)}, a_{11(1-f)}] & \vdots & [a_{11(t)}, a_{11(1-f)}] & [a_{11(t)}, a_{11(1-f)}] & \vdots & [a_{11(t)}, a_{11(1-f)}] & [a_{11(t)}, a_{11(1-f)}] \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ [a_{11(t)}, a_{11(1-f)}] & [a_{11(t)}, a_{11(1-f)}] & \vdots & [a_{11(t)}, a_{11(1-f)}] & [a_{11(t)}, a_{11(1-f)}] & \vdots & [a_{11(t)}, a_{11(1-f)}] & [a_{11(t)}, a_{11(1-f)}] \\ [a_{11(t)}, a_{11(1-f)}] & [a_{11(t)}, a_{11(1-f)}] & \vdots & [a_{11(t)}, a_{11(1-f)}] & [a_{11(t)}, a_{11(1-f)}] & \vdots & [a_{11(t)}, a_{11(1-f)}] & [a_{11(t)}, a_{11(1-f)}] \end{bmatrix}$$

$= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix}$ is a square vague block matrix since all A_{ij}'s are square blocks.

Definition 3.8: If a square vague block matrix is such that the blocks $A_{ij} = [[0,0]]$ for all $i \neq j$ then the vague matrix A is said to be a diagonal vague block matrix. Thus $\begin{bmatrix} A_{11} & 0 & 0 \\ 0 & A_{12} & 0 \end{bmatrix}$, where $0 = [[0,0]]$ is a diagonal vague block matrix.

Theorem 3.9: If $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{n\times p}$ are two vague matrices such that $AB = C = [c_{ij}]_{m\times p}$ then the j-th column of C is AB_j, where $B_j = \begin{bmatrix} [b_{1j(t)}, b_{1j(1-f)}] \\ [b_{2j(t)}, b_{2j(1-f)}] \\ \vdots \\ [b_{nj(t)}, b_{nj(1-f)}] \end{bmatrix}$ are the column partition of vague matrix B.

Proof: Let vague matrix B of order $n \times p$ be partition into p column vectors $(n \times 1)$ vague matrices as $B = [B_1 \quad B_2 \quad \cdots \quad B_j \quad \cdots \quad B_p]$ where j=1,2,3…,p. to find a column of the product AB. From the product rule of the vague matrices, the elements of the product is, $c_{ij} = \left[[\max_k(\min\{a_{ik(t)}, b_{ik(t)}\}, \max_k(\min\{a_{ik(1-f)}, b_{ik(1-f)}\}]\right], i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$, where $A = \left[[a_{ik(t)}, a_{ik(1-f)}]\right]_{m\times n}, B = \left[[b_{ik(t)}, b_{ik(1-f)}]\right]_{n\times p}, C = \left[[c_{ik(t)}, c_{ik(1-f)}]\right]_{m\times n}$ and C=AB.

Therefore j-th column of C is obtained by giving the values 1,2…,m to I and it is,

$$C_j = \begin{bmatrix} [\max_k(\min\{a_{1k(t)}, b_{kj(t)}\}, \max_k(\min\{a_{1k(1-f)}, b_{kj(1-f)}\}] \\ [\max_k(\min\{a_{2k(t)}, b_{kj(t)}\}, \max_k(\min\{a_{2k(1-f)}, b_{kj(1-f)}\}] \\ \vdots \\ [\max_k(\min\{a_{mk(t)}, b_{kj(t)}\}, \max_k(\min\{a_{mk(1-f)}, b_{kj(1-f)}\}] \end{bmatrix}$$

 ISSN 2349-6193
Impact Factor: 3.866
IJESMR
International Journal OF Engineering Sciences & Management Research

$$= \begin{bmatrix} [a_{11(t)}, a_{11(1-f)}] & [a_{12(t)}, a_{12(1-f)}] & \vdots & [a_{1n(t)}, a_{1n(1-f)}] \\ [a_{21(t)}, a_{21(1-f)}] & [a_{22(t)}, a_{22(1-f)}] & \vdots & [a_{2n(t)}, a_{2n(1-f)}] \\ \cdots & \cdots & \cdots & \cdots \\ [a_{m1(t)}, a_{m1(1-f)}] & [a_{m2(t)}, a_{m2(1-f)}] & \vdots & [a_{mn(t)}, a_{mn(1-f)}] \end{bmatrix} \begin{bmatrix} [b_{1j(t)}, b_{1j(1-f)}] \\ [b_{2j(t)}, b_{2j(1-f)}] \\ \vdots \\ [b_{nj(t)}, b_{nj(1-f)}] \end{bmatrix}, j = 1,2,\ldots,p.$$

$= AB_j,\ j = 1,2,\ldots,p$. Hence the theorem

Theorem 3.10: Let A be an $m \times n$ vague matrix and B be an $n \times p$ vague matrix. Let B(or A) be partitioned into two blocks by column partitioning only. Then the product AB is also partitioned into two blocks of same column(row) partitioning.

Proof: Let $B = [B_1 \quad B_2]$ where B_1 is of order $n \times t$ and B_2 is of order $n \times (p-t)$ vague matrix. Then

$$AB = A[b_1 \quad b_2 \quad \cdots \quad b_t \quad \vdots \quad b_{(t+1)} \quad \cdots \quad b_p] \quad \text{where} \quad b_j = \begin{bmatrix} [b_{1j(t)}, b_{1j(1-f)}] \\ [b_{2j(t)}, b_{2j(1-f)}] \\ \vdots \\ [b_{nj(t)}, b_{nj(1-f)}] \end{bmatrix},$$

$= [Ab_1 \quad Ab_2 \quad \cdots \quad Ab_t \quad \vdots \quad Ab_{(t+1)} \quad \cdots \quad Ab_p] = [AB_1 \quad \vdots \quad AB_2]$. Hence the proof.

Operations on vague block matrix 3.11:

Addition: The conformal vague matrices can be added by block as addition of two vague matrices of same dimensions.

$$A + B = \begin{bmatrix} A_{11} + B_{11} & A_{12} + B_{12} & \vdots & A_{1q} + B_{1q} \\ A_{21} + B_{21} & A_{22} + B_{22} & \vdots & A_{2q} + B_{2q} \\ \cdots & \cdots & \cdots & \cdots \\ A_{p1} + B_{p1} & A_{p2} + B_{p2} & \vdots & A_{pq} + B_{pq} \end{bmatrix}$$

Scalar multiplication: In scalar multiplication the vague block matrix is multiplied by a scalar. That is each block of partition vague matrix is multiplied by scalar. That is, $\alpha A = \begin{bmatrix} \alpha A_{11} & \alpha A_{12} & \vdots & \alpha A_{1q} \\ \alpha A_{21} & \alpha A_{22} & \vdots & \alpha A_{2q} \\ \cdots & \cdots & \cdots & \cdots \\ \alpha A_{p1} & \alpha A_{p2} & \vdots & \alpha A_{pq} \end{bmatrix}$.

Theorem 3.12: If AB=C the vague submatrix containing rows $i_1, i_2, \ldots, i_r$ and $j_1, j_2, \ldots, j_s$ of C is equal to the product of the vague submatrix with these rows of A and the vague submatrix with these columns of B.

Proof: Let $A = [[a_{ij(t)}, a_{ij(1-f)}]]_{m\times n}$ and $B = [[b_{ij(t)}, b_{ij(1-f)}]]_{n\times p}$ be two vague matrices. Then

$$C = [[c_{ij(t)}, c_{ij(1-f)}]] = \left[[\max_{j=1}^{n}(\min\{a_{ij(t)}, b_{jk(t)}\}, \max_{j=1}^{n}(\min\{a_{ij(1-f)}, b_{jk(1-f)}\}] \right],$$ where i=1,2,…,m and k=1,2,…,p. Now the vague submatrix C_{ik} of vague matrix C with rows $i_1, i_2, \ldots, i_r$ and columns $j_1, j_2, \ldots, j_s$ is obtained by replacing row i and column k of C by these rows and columns. It is

$$C_{ik} = \left[[\max_{j=1}^{n}(\min\{a_{ij(t)}, b_{jk(t)}\}, \max_{j=1}^{n}(\min\{a_{ij(1-f)}, b_{jk(1-f)}\}] \right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1),$$

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where $i = i_1, i_2, ..., i_r$ and columns $k = j_1, j_2, ..., j_s$. Again the product of the given vague submatrices A_i and B_k of vague matrix A nad B respectively is

$$A_i B_k = \begin{bmatrix} [a_{i_1 1(t)}, a_{i_1 1(1-f)}] & [a_{i_1 2(t)}, a_{i_1 2(1-f)}] & \cdots & [a_{i_1 n(t)}, a_{i_1 n(1-f)}] \\ [a_{i_2 1(t)}, a_{i_2 1(1-f)}] & [a_{i_2 2(t)}, a_{i_2 2(1-f)}] & \cdots & [a_{i_2 n(t)}, a_{i_2 n(1-f)}] \\ \vdots & \vdots & \vdots & \vdots \\ [a_{i_r 1(t)}, a_{i_r 1(1-f)}] & [a_{i_r 2(t)}, a_{i_r 2(1-f)}] & \cdots & [a_{i_r n(t)}, a_{i_r n(1-f)}] \end{bmatrix}_{r \times n} \begin{bmatrix} [b_{1j_1(t)}, b_{1j_1(1-f)}] & [b_{1j_1(t)}, b_{1j_1(1-f)}] & \cdots & [b_{1j_1(t)}, b_{nj_1(1-f)}] \\ [b_{2j_2(t)}, b_{2j_2(1-f)}] & [b_{2j_2(t)}, b_{2j_2(1-f)}] & \cdots & [b_{2j_2(t)}, b_{2j_2(1-f)}] \\ \vdots & \vdots & \vdots & \vdots \\ [b_{nj_r(t)}, b_{nj_r(1-f)}] & [b_{nj_r(t)}, b_{nj_r(1-f)}] & \cdots & [b_{nj_r(t)}, b_{nj_r(1-f)}] \end{bmatrix}_{n \times s}$$

$$= \begin{bmatrix} [\max_{j=1}^{n}(\min\{a_{ij(t)}, b_{jk(t)}\}, \max_{j=1}^{n}(\min\{a_{ij(1-f)}, b_{jk(1-f)}\}] & \cdots & [\max_{j=1}^{n}(\min\{a_{ij(t)}, b_{jk(t)}\}, \max_{j=1}^{n}(\min\{a_{ij(1-f)}, b_{jk(1-f)}\}] \\ [\max_{j=1}^{n}(\min\{a_{ij(t)}, b_{jk(t)}\}, \max_{j=1}^{n}(\min\{a_{ij(1-f)}, b_{jk(1-f)}\}] & \cdots & [\max_{j=1}^{n}(\min\{a_{ij(t)}, b_{jk(t)}\}, \max_{j=1}^{n}(\min\{a_{ij(1-f)}, b_{jk(1-f)}\}] \\ \vdots & \vdots & \vdots_r \\ [\max_{j=1}^{n}(\min\{a_{ij(t)}, b_{jk(t)}\}, \max_{j=1}^{n}(\min\{a_{ij(1-f)}, b_{jk(1-f)}\}] & \cdots & [\max_{j=1}^{n}(\min\{a_{ij(t)}, b_{jk(t)}\}, \max_{j=1}^{n}(\min\{a_{ij(1-f)}, b_{jk(1-f)}\}] \end{bmatrix}$$

$$= \left[[\max_{j=1}^{n}(\min\{a_{ij(t)}, b_{jk(t)}\}, \max_{j=1}^{n}(\min\{a_{ij(1-f)}, b_{jk(1-f)}\}] \right]_{r \times s} \quad ....(2)$$

where $i = i_1, i_2, ..., i_r$ and columns $k = j_1, j_2, ..., j_s$. Therefore the relation (1) and (2) together gives the results.

Vague direct sum:

Definition 3.14: Let $A_1, A_2, ..., A_r$ be square vague matrices of orders $m_1, m_2, ..., m_r$ respectively. The diagonal vague matrix,

$$\mathrm{diag}\,(A_1, A_2, ..., A_r) = \begin{bmatrix} A_1 & 0 & \cdots & 0 \\ 0 & A_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_r \end{bmatrix}_{(m_1 + m_2 + ... + m_r)}$$

is called the vague direct sum of the square vague matrices $A_1, A_2, ..., A_r$ and is expressed by $A_1 \oplus A_2 \oplus ...... \oplus A_r$ of order $(m_1 + m_2 + ... + m_r)$. It is also called the vague block diagonalize form.

Properties of vague direct sum:
Vague direct sum of vague matrices possesses the following algebraic properties:

1. Commutative property: Commutative property does not hold of the square vague matrices.
   Let A and B be two square vague matrices. Then the vague direct sum of A and B are $A \oplus B = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ and $B \oplus A = \begin{bmatrix} B & 0 \\ 0 & A \end{bmatrix}$. It is obvious that, $A \oplus B \neq B \oplus A$.

ISSN 2349-6193
Impact Factor: 3.866

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2. Associative property: Let A, B and C be three square vague matrices. Then $A\oplus B=\begin{bmatrix}A & 0\\ 0 & B\end{bmatrix}=D$.

   Now,

   $(A\oplus B)\oplus C=D\oplus C=\begin{bmatrix}D & 0\\ 0 & C\end{bmatrix}=\begin{bmatrix}A & 0 & 0\\ 0 & B & 0\\ 0 & 0 & C\end{bmatrix}$. Similarly, $B\oplus C=\begin{bmatrix}B & 0\\ 0 & C\end{bmatrix}=E$. Now,

   $A\oplus(B\oplus C)=A\oplus E=\begin{bmatrix}A & 0\\ 0 & E\end{bmatrix}=\begin{bmatrix}A & 0 & 0\\ 0 & B & 0\\ 0 & 0 & C\end{bmatrix}$. Therefore,

   $(A\oplus B)\oplus C=A\oplus(B\oplus C)$. Hence, the associative law holds for direct sum of the vague block matrix.

3. Mixed sum: $(A+B)\oplus(C+D)=(A\oplus B)+(C\oplus D)$, if the addition are conformable corresponding vague block matrices. By the definition of the vague block matrix and vague matrix addition,

   $$(A+B)\oplus(C+D)=\begin{bmatrix}(A+B) & 0\\ 0 & (C+D)\end{bmatrix}=\begin{bmatrix}A & 0\\ 0 & C\end{bmatrix}+\begin{bmatrix}B & 0\\ 0 & D\end{bmatrix}=(A\oplus C)+(B\oplus D)$$

4. Vague matrix multiplication of vague direct sum: $(A\oplus B)(C\oplus D)=(AC)\oplus(BD)$ if the multiplication is conformable for vague matrix.

   $$(A\oplus B)(C\oplus D)=\begin{bmatrix}A & 0\\ 0 & B\end{bmatrix}+\begin{bmatrix}C & 0\\ 0 & D\end{bmatrix}=\begin{bmatrix}AC & 0\\ 0 & BD\end{bmatrix}(AC)\oplus(BD)$$

5. Transposition of vague matrices: $(A\oplus B)^T=A^T\oplus B^T$. Since, $A\oplus B=\begin{bmatrix}A & 0\\ 0 & B\end{bmatrix}$ then

   $$(A\oplus B)^T=\begin{bmatrix}A & 0\\ 0 & B\end{bmatrix}^T=\begin{bmatrix}A^T & 0\\ 0 & B^T\end{bmatrix}=A^T\oplus B^T.$$

Vague Kronecker product of vague matrices 3.15:

Let $A=\left[\left\langle a_{ij(t)},a_{ij(1-f)}\right\rangle\right]_{m\times n}$ and $B=\left[\left\langle b_{ij(t)},b_{ij(1-f)}\right\rangle\right]_{p\times q}$ be two rectangular vague matrices. Then the vague Kronecker product of A and B, denoted by $A\otimes B$ is defined as the partitioned vague matrix

$A\otimes B=\begin{bmatrix}a_{11}B & a_{12}B & \cdots & a_{1n}B\\ a_{21}B & a_{22}B & \cdots & a_{2n}B\\ \vdots & \vdots & \vdots & \vdots\\ a_{m1}B & a_{m2}B & \cdots & a_{mn}B\end{bmatrix}_{mp\times nq}$ where $a_{ij}=[<a_{ij(t)},a_{ij(1-f)}>]$ for i=1,2,….,m and j=1,2,….n. It has mn blocks. The ij th blocks $a_{ij}B$ of order $p\times q$.

Vague Kronecker product of two vague column vectors 3.16:

Let $x=[<x_{1(t)},x_{1(1-f)}><x_{2(t)},x_{2(1-f)}>..............<x_{n(t)},x_{n(1-f)}>]^T$ and $y=[<y_{1(t)},y_{1(1-f)}><y_{2(t)},y_{2(1-f)}>..............<y_{n(t)},y_{n(1-f)}>]^T$ be two column vague vectors. Then by definition of vague kronecker product, we have

ISSN 2349-6193
Impact Factor: 3.866
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$$x \otimes y = \begin{bmatrix} [<x_{1(t)}, x_{1(1-f)}>]y \\ [<x_{2(t)}, x_{2(1-f)}>]y \\ \vdots \\ [<x_{n(t)}, x_{n(1-f)}>]y \end{bmatrix} \begin{bmatrix} [<x_{1(t)}, x_{1(1-f)}>][<y_{1(t)}, y_{1(1-f)}>] \\ \vdots \\ [<x_{1(t)}, x_{1(1-f)}>][<y_{m(t)}, y_{m(1-f)}>] \\ [<x_{2(t)}, x_{2(1-f)}>][<y_{1(t)}, y_{1(1-f)}>] \\ \\ [<x_{2(t)}, x_{2(1-f)}>][<y_{m(t)}, y_{m(1-f)}>] \\ \\ [<x_{n(t)}, x_{n(1-f)}>][<y_{1(t)}, y_{1(1-f)}>] \\ \\ [<x_{n(t)}, x_{n(1-f)}>][<y_{m(t)}, y_{m(1-f)}>] \end{bmatrix}_{nm \times 1}$$

Properties of vague Kronecker product 3.17:

Let A, B and C be vague matrices then the vague Kronecker product satisfies the following:

1. Commutative : The vague Kronecker product is not commutative, $A \otimes B \neq B \otimes A$.
2. Distributive: If B and C are conformable for addition, then
   $A \otimes (B + C) = A \otimes B + A \otimes C$,[left distribution]
   $(B + C) \otimes A = B \otimes A + C \otimes A$,[right distribution]
3. Associative: $A \otimes (B \otimes C) = (A \otimes B) \otimes C$.
4. Transposition: $(A \otimes B)^T = A^T \otimes B^T$.
5. Trace: $Tr(A \otimes B) = (TrA)(TrB)$.
6. Two vague column vectors α and β, not necessarily of the same order: $\alpha^T \otimes \beta = \beta\alpha^T = \beta \otimes \alpha^T$.
7. $\det(A_{m\times m} \otimes B_{n\times n}) = (\det A)^m (\det B)^n$.

Vague Kronecker sum 3.18:

The vague Kronecker sum of two square vague matrices $A_{n\times n}$ and $B_{m\times m}$ is defined by $A \dagger B = A \otimes I_m + I_n \otimes B$, which is an $nm \times nm$ vague matrix.

Example 3.19: Let $A = \begin{bmatrix} [0.3,0.5] & [0.3,0.6] & [0.1,0.8] \\ [0.2,0.6] & [0.4,0.6] & [0.5,0.6] \\ [0.1,0.7] & [0.2,0.5] & [0.3,0.6] \end{bmatrix}$ and $B = \begin{bmatrix} [0.1,0.7] & [0.4,0.5] \\ [0.2,0.5] & [0.1,0.6] \end{bmatrix}$ be two vague matrices. Then $A \dagger B = A \otimes I_2 + I_3 \otimes B$.

$$A \dagger B = \begin{bmatrix} [0.3,0.7] & [0.4,0.5] & \vdots & [0.3,0.6] & [0,0] & \vdots & [0.1,0.8] & [0,0] \\ [0.2,0.5] & [0.3,0.6] & \vdots & [0,0] & [0.3,0.6] & \vdots & [0,0] & [0.1,0.8] \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ [0.2,0.6] & [0,0] & \vdots & [0.4,0.7] & [0.4,0.5] & \vdots & [0.5,0.6] & [0,0] \\ [0,0] & [0.2,0.6] & \vdots & [0.2,0.5] & [0.4,0.6] & \vdots & [0,0] & [0.5,0.6] \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ [0.1,0.7] & [0,0] & \vdots & [0.2,0.5] & [0,0] & \vdots & [0.3,0.7] & [0.4,0.5] \\ [0,0] & [0.1,0.7] & \vdots & [0,0] & [0.2,0.5] & \vdots & [0.2,0.5] & [0.3,0.6] \end{bmatrix}$$

ISSN 2349-6193
Impact Factor: 3.866

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Some relational operations on vague block matrices:

Here we define four special types of reflexivity and irreflexivity of a vague matrices.

Definition 3.20: Let A be a vague matrices of any order then,

(i) R_1: A is of type-1 reflexive if $a_{ij(t)} = 1$ and $a_{ij(1-f)} = 1$, for all i=1,2,…,n.

(ii) R_2: A is of type-2 reflexive if $(a_{ii(1-f)} \wedge a_{jj(1-f)}) \geq a_{ij(1-f)}$, for all i, j=1,2,…,n.

(iii) R_3: A is of type-3 reflexive if $(a_{ii(t)} \wedge a_{jj(t)}) \geq a_{ij(t)}$, for all i, j=1,2,…,n.

(iv) R_4: A is of type-4 reflexive if $(a_{ii(1-f)} \wedge a_{jj(1-f)}) \geq a_{ij(1-f)}$ and $(a_{ii(t)} \wedge a_{jj(t)}) \geq a_{ij(t)}$, where i, j=1,2,…,n.

For irreflexivity,

(i) R_1: A is of type-1 reflexive if $a_{ij(t)} = 0$ and $a_{ij(1-f)} = 0$, for all i=1,2,…,n.

(ii) R_2: A is of type-2 reflexive if $(a_{ii(1-f)} \vee a_{jj(1-f)}) \leq a_{ij(1-f)}$, for all i, j=1,2,…,n.

(iii) R_3: A is of type-3 reflexive if $(a_{ii(t)} \vee a_{jj(t)}) \leq a_{ij(t)}$, for all i, j=1,2,…,n.

(iv) R_4: A is of type-4 reflexive if $(a_{ii(1-f)} \vee a_{jj(1-f)}) \leq a_{ij(1-f)}$ and $(a_{ii(t)} \vee a_{jj(t)}) \leq a_{ij(t)}$, where i, j=1,2,…,n.

Theorem 3.21: If vague matrices A and B be reflexive of any type then direct sum of these vague matrices is also reflexive of the same type.

Proof: (i) Let vague matrices A and B be type-1 reflexive, then $[<a_{ii(t)}, a_{ii(1-f)}>] = [<1,1>]$ and $[<b_{ii(t)}, b_{ii(1-f)}>] = [<1,1>]$. Then the direct sum of these vague matrices A and B be vague block matrix, $S = A \oplus B = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$. Now $[<s_{ii(t)}, s_{ii(1-f)}>] = [<1,1>]$, since diagonal elements in vague block matrices S are vague matrices A and B and diagonal elements in A and B are [<1,1>]. Hence the direct sum S of the vague matrices A and B is type-1 reflexive.

(ii) Let vague matrices A and B be type-2 reflexive, then $(a_{ii(1-f)} \wedge a_{jj(1-f)}) \geq a_{ij(1-f)}$ and $(b_{ii(1-f)} \wedge b_{jj(1-f)}) \geq b_{ij(1-f)}$. Then the direct sum of these vague matrices A and B be vague block matrix $S = A \oplus B$. Now for A blocks we have

$$s_{ij(1-f)} = a_{ij(1-f)} \; [i = 1,2,....,m, \; j = 1,2,....,m]$$
$$\leq a_{ii(1-f)} \wedge a_{jj(1-f)} \; \text{[as A is type- 3 reflexive]}$$
$$= s_{ii(1-f)} \wedge s_{jj(1-f)}.$$

Now for B blocks we have,

$$s_{(m+p)(m+q)(1-f)} = b_{pq(1-f)} \; [p = 1,2,....,n, \; q = 1,2,....,n]$$
$$\leq b_{pp(1-f)} \wedge b_{qq(1-f)} \; \text{[as B is type- 3 reflexive]}$$
$$= s_{(m+p)(m+p)(1-f)} \wedge s_{(m+q)(m+q)(1-f)}. \text{[as off diagonal blocks are vague zero matrices]}$$

Therefore, $s_{kl(1-f)} \leq s_{kk(1-f)} \wedge s_{ll(1-f)} \; k = 1,2,...,m+n, \; l = 1,2,.......,m+n.$ Hence the direct sum S of the vague matrices A and B is type-3 reflexive.

(iii) Let vague matrices A and B be type-3 reflexive, then $(a_{ii(t)} \wedge a_{jj(t)}) \geq a_{ij(t)}$ and $(b_{ii(t)} \wedge b_{jj(t)}) \geq b_{ij(t)}$. Then the direct sum of these vague matrices A and B be vague block matrix $S = A \oplus B$. Now for A blocks we have

 ISSN 2349-6193
Impact Factor: 3.866

IJESMR
International Journal OF Engineering Sciences & Management Research

$$s_{ij(t)} = a_{ij(t)} \; [i=1,2,....,m, \; j=1,2,....,m]$$
$$\le a_{ii(t)} \wedge a_{jj(t)} \; [\text{as A is type- 3 reflexive }]$$
$$= s_{ii(t)} \wedge s_{jj(t)}.$$

Now for B blocks we have,

$$s_{(m+p)(m+q)(t)} = b_{pq(t)} \; [p=1,2,....,n, \; q=1,2,....,n]$$
$$\le b_{pp(t)} \wedge b_{qq(t)} \; [\text{as B is type- 3 reflexive }]$$
$$= s_{(m+p)(m+p)(t)} \wedge s_{(m+q)(m+q)(t)} .[\text{as off diagonal blocks are vague zero matrices}]$$

Therefore, $s_{kl(t)} \le s_{kk(t)} \wedge s_{ll(t)}$ $k=1,2,...,m+n,$ $l=1,2,.......,m+n.$ Hence the direct sum S of the vague matrices A and B is type-3 reflexive.

(iv) Let vague matrices A and B be type-4 reflexive, then $a_{ii(1-f)} \wedge a_{jj(1-f)}$, $a_{ii(t)} \wedge a_{jj(t)}$ and $b_{ii(1-f)} \wedge b_{jj(1-f)}$, $b_{ii(t)} \wedge b_{jj(t)}$. Then the direct sum of these vague matrices A nad B be vague block matrix of type-4 reflexive, by results (ii) and (iii). The direct sum of the vague matrices reflexive of any type is also reflexive of the same type.

Theorem 3.22: If the vague matrices A and B be type-1 reflexive then the vague Kronecker product of these vague matrices is also type-1 reflexive.

Proof: Let vague matrices A and B be type-1 reflexive, then $[<a_{ii(t)}, a_{ii(1-f)}>] = [<1,1>]$ and $[<b_{ii(t)}, b_{ii(1-f)}>] = [<1,1>]$. Then the vague Kronecker product of these vague matrices A and B be vague block matrix,

$$S = A \otimes B = \begin{bmatrix} a_{11}B & a_{11}B & \cdots & a_{11}B \\ a_{11}B & a_{11}B & \cdots & a_{11}B \\ \cdots & \cdots & \ddots & \cdots \\ a_{11}B & a_{11}B & \cdots & a_{11}B \end{bmatrix}$$

Here $a_{ii} = [1,1]$ for all i=1,2,….,m as A is a vague matrix of type-1 reflexive and diagonal elements of B are $b_{jj} = [1,1]$ for all j=1,2….,n as B is a vague matrix of type-1 reflexive. Therefore, $[<s_{pp(t)}, s_{pp(1-f)}>] = [<1,1>]$ for p=1,2,….,mn, where m and n are the order of the vague matrices A nad B respectively. Hence the vague Kronecker product of vague matrices of type-1 reflexive is also type-1 reflexive.

Theorem 3.23: If the vague matrices A and B be type-2 reflexive then the vague Kronecker product of these vague matrices is also type-2 reflexive.

Proof: Let vague matrices A and B be type-2 reflexive, then $a_{ii(1-f)} \wedge a_{jj(1-f)} \ge a_{ij(1-f)}$ and $b_{ii(1-f)} \wedge b_{jj(1-f)} \ge b_{ij(1-f)}$. Then the vague Kronecker product of these vague matrices A nad B be vague block matrix $S = A \otimes B$. Vague block matrix S contains mm blocks, diagonal blocks are vague matrices $a_{ii}B$ and off diagonal blocks are $a_{jj}B$ where $i \ne j$.

Now for the diagonal blocks,

$$s_{pq(1-f)} = \min\{a_{ii(1-f)}, b_{pq(1-f)}\} \; [\text{where } i=1,2,….,m \text{ and } p,q=1,2,…,n]$$
$$\le \min\{a_{ii(1-f)}, b_{pp(1-f)} \vee b_{qq(1-f)}\} \; [\text{as B is type-3 reflexive}]$$
$$\le s_{pp(1-f)} \vee s_{qq(1-f)}.$$

Now for off diagonal blocks,

ISSN 2349-6193
Impact Factor: 3.866

$$s_{pq(1-f)} = \min\{a_{ij(1-f)}, b_{pq(1-f)}\} \text{[where i, j=1,2,….,m and p,q=1,2,…,n]}$$
$$\leq \min\{a_{ij(t)}, b_{pp(1-f)} \vee b_{qq(1-f)}\} \text{[as B is type-3 reflexive]}$$
$$\leq s_{pp(1-f)} \vee s_{qq(1-f)}.$$

Again, $a_{ii(1-f)} \wedge a_{jj(1-f)} \geq a_{ij(1-f)}$ for all i, j= 1,2,…,m. Therefore, $s_{kl(1-f)} \leq s_{kk(1-f)} \vee s_{ll(1-f)}$, k=1,2,……,mn, l=1,2,….,mn. Hence the vague Kronecker product of the vague matrices A and B is also type-3 reflexive.

Theorem 3.24: If the vague matrices A and B be type-3 reflexive then the vague Kronecker product of these vague matrices is also type-3 reflexive.

Proof: Let vague matrices A and B be type-3 reflexive, then $a_{ii(t)} \wedge a_{jj}(t) \geq a_{ij(t)}$ and $b_{ii(t)} \wedge b_{jj}(t) \geq b_{ij(t)}$. Then the vague Kronecker product of these vague matrices A and B be vague block matrix $S = A \otimes B$. Vague block matrix S contains mm blocks, diagonal blocks are vague matrices $a_{ii}B$ and off diagonal blocks are $a_{jj}B$ where $i \neq j$.

Now for the diagonal blocks,

$$s_{pq(t)} = \min\{a_{ii(t)}, b_{pq(t)}\} \text{[where i=1,2,….,m and p,q=1,2,…,n]}$$
$$\leq \min\{a_{ii(t)}, b_{pp(t)} \vee b_{qq(t)}\} \text{[as B is type-3 reflexive]}$$
$$\leq s_{pp(t)} \vee s_{qq(t)}.$$

Now for off diagonal blocks,

$$s_{pq(t)} = \min\{a_{ij(t)}, b_{pq(t)}\} \text{[where i, j=1,2,….,m and p,q=1,2,…,n]}$$
$$\leq \min\{a_{ij(t)}, b_{pp(t)} \vee b_{qq(t)}\} \text{[as B is type-3 reflexive]}$$
$$\leq s_{pp(t)} \vee s_{qq(t)}.$$

Again, $a_{ii(t)} \wedge a_{jj(t)} \geq a_{ij(t)}$ for all i, j= 1,2,…,m. Therefore, $s_{kl(t)} \leq s_{kk(t)} \vee s_{ll(t)}$, k=1,2,……,mn, l=1,2,….,mn. Hence the vague Kronecker product of the vague matricies A and B is also type-3 reflexive.

Theorem 3.25: If vague matrices A and B be of type-4 reflexive then the vague kronecker product of these vague matrices is also type-4 reflexive.

Proof: Let vague matrices A and B be type-4 reflexive, that is $a_{ii(t)} \wedge a_{jj}(t) \geq a_{ij(t)}$, $a_{ii(1-f)} \wedge a_{jj(1-f)} \geq a_{ij(1-f)}$ and $b_{ii(1-f)} \wedge b_{jj(1-f)} \geq b_{ij(1-f)}$. $b_{ii(t)} \wedge b_{jj}(t) \geq b_{ij(t)}$. Then from the above two theorems. Vague Kronecker product of these vague matrices A and B be vague block matrix which is type-4 reflexive.

Theorem 3.26: If vague matrices A and B be type-1 then vague Kronecker sum of these vague matrices is also type-1 reflexive.

Proof: Let vague matrices A and B of order $m \times m$ and $n \times n$ respectively be type-1 reflexive, then $[< a_{ii(t)}, a_{ii(1-f)} >] = [< 1,1 >]$ and $[< b_{ii(t)}, b_{ii(1-f)} >] = [< 1,1 >]$. Then the vague Kronecker sum of these vague matrices A and B be vague block matrix,

$$S = A \dagger B = (A \otimes I_n) + (I_m \otimes B)$$

where I_n and I_m are the vague identity matrices. Now vague identity matrices I_n and I_m are type-1 reflexive vague matrices. Therefore, by the theorem, direct sum of type-1 reflexive vague matrices, is again type-1 reflexive. Hence vague Kronecker sum of vague matrices of type-1 reflexive is also of type-1 reflexive.

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ISSN 2349-6193
Impact Factor: 3.866
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